Angular distribution of Cerenkov light from electrons both produced and stopping in water

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Abstract. The Goudsmit-Saunderson treatment of multiple scattering is used to calculate the angular distribution of Cerenkov light produced by electrons which are wholly contained in water. The calculations are completely independent of any Monte Carlo method and the results are in remarkably good agreement with those from EGS4.

1. Introduction

In SNOman electron tracks are generated with EGS4 [1] and emission of Cerenkov light is patched in segment by segment. The correct angular distribution of Cerenkov light — important for vertex finding and for distinguishing charged from neutral current events — is dependent on the proper handling of multiple scattering within EGS4. This note explains how the angular distribution of Cerenkov light has been calculated with a method which is (very nearly) exact, providing a validation of the EGS4 code in this application.

2. Exact calculation of the angular distribution.

The angular distribution of electrons, after any specified path length, can be calculated exactly for all angles (0–180°) in terms of simple functions. This is an ancient result due to Goudsmit and Saunderson [2].

If $f(t,\theta)d\cos\theta$ is the fraction of electrons to be found within the interval $d\cos\theta$ after traversing a path length t, then $f(t,\theta)$ is given exactly by

$$f(t,\theta) = \sum_l \left(l + \frac{1}{2}\right) P_l(\cos\theta) \exp\left\{-Nt\int \sigma(\chi)[1 - P_l(\cos\chi)]d\cos\chi\right\} \qquad (1)$$

where N is the density of scattering centres and $\sigma(\chi)$ the scattering cross section. It must be emphasised that t is the total path length, taking into account all the twists and turns of the electron due to scattering, and not the projection on the initial direction. For many applications this is inconvenient, but it is ideal for the problem in hand.

As it stands, (1) is only valid if $\sigma(\chi)$ is independent of the path length t. For an electron both produced and stopping in water, $\sigma(\chi)$ increases rapidly as the electron slows down. The necessary modification is trivial [3]

$$Nt\int\sigma(\chi)[1-P_l(\cos\chi)]d\cos\chi \rightarrow N\int_0^t dt\int\sigma(t,\chi)[1-P_l(\cos\chi)]d\cos\chi \quad (2)$$

and an element of path length dt can be replaced by an element of electron kinetic energy dT via

$$dt=\frac{dt}{dT}dT; \quad \frac{dT}{dt}\sim -1.65\left(\frac{c}{v}\right)^2 \text{ MeV g}^{-1}\text{ cm}^2 \quad (3)$$

The cross section $\sigma(T,\chi)$ is just the Rutherford cross section, with a correction for screening. The integral over $\cos\chi$ yields [4]

$$N\int\sigma(T,\chi)[1-P_l(\cos\chi)]d\cos\chi=\frac{1}{2}\chi_c^2 l(l+1)\left[ln\frac{2}{\chi_a}-\frac{1}{2}-\left(\frac{1}{2}+\frac{1}{3}\cdots\frac{1}{l}\right)\right] \quad (4)$$

where

$$\chi_c^2=4\pi Ne^4Z(Z+1)/(pv)^2 \quad (5)$$

where Z is the nuclear charge and p,v are the momentum and speed of the electron. The quantity χ_c^2 is thus a function of T or t. (Note that the path length travelled has been removed from the definition of χ_c^2 given in [4].)

The quantity χ_a is the screening angle and is approximately λ/a, where a is the Fermi radius of the atom and $\lambda=\hbar/p$.

The fraction $f(t,\theta)$ is easy to calculate provided that only a relatively small number of Legendre polynomials $P_l(\cos\theta)$ contribute significantly in (1). For a root mean square scattering angle of $\sim 50°$ it is adequate to extend the sum to no more than $l=5$. For a root mean square scattering angle $\sim 30°$ it is necessary to go as far as $l=10$. For $\sim 5°$ $l=50$ in fact suffices.

3. Implementation

For track lengths greater than that corresponding to a root mean square scattering angle of 5° the Goudsmit- Saunderson (GS) method was used, eqs. (1)–(5), with the series terminating at $l=100$ or where the coefficients of the polynomials drop below 10^{-5}. The screening angle was defined by

$$\frac{2}{\chi_a}=0.858\frac{137p}{mc} \quad (6)$$

and in place of the approximate expression (3) for energy loss

$$\frac{dT}{dt} = -\left(\frac{c}{v}\right)^2 \left[1.65 + 0.1973\ lnT + 0.01276(lnT)^2\right] \tag{7}$$

where T is in MeV. The expression (7) accurately matches the stopping power data for electrons in water given in [5], between 0.25 MeV and 15 MeV.

For shorter track lengths, corresponding to root mean square scattering angles less than 5°, the full GS method becomes tedious and eventually impossible to implement. Here the first (pure gaussian) term in Moliere's formula (see [4]) was used with

$$< \theta^2 >= \chi_c^2 t B \tag{8}$$

The Moliere parameter B is given by [4]

$$B - lnB = b \tag{9a}$$

and

$$e^b = \frac{6}{7}\frac{\chi_c^2}{\chi_a^2} t = 6680t/\beta^2 \tag{9b}$$

The parameter B was approximated by the expression

$$B = \frac{10}{9} b + \frac{13}{9} \tag{9c}$$

which is a good representation of (9a) in the range $b = 3 - 11$; $B = 5 - 13$. The quantity e^b is approximately the number of scatters (which is finite because of screening) in path length t. For $b << 3$ the path length is so short and the angular distribution so narrow that the representation (9c) is as good as any — these conditions only obtain over a very small portion of track at the very beginning. (The gaussian generated by (8), (9) matches well the GS solution at small angles for $< \theta^2 >^{1/2} = 10° - 30°$).

The calculation proceeded in small steps of T (the steps can be made as small as desired at the cost only of time). At each step the changes in the polynomial coefficients were calculated and the the values updated; the same was done for the gaussian mean square scattering angle. The angular distribution then being available, each small segment of track contributed an amount of light proportional to the length of the segment, multiplied by the speed dependent factor $1 - (\frac{c}{nv})^2$, at the local Cerenkov angle

$$\theta_c = \cos^{-1}\left(\frac{c}{nv}\right)$$

where n is the refractive index of water (taken as 1.33).

The rest is geometry. When the track segment makes an angle θ with the original direction, the fraction of Cerenkov light emitted at angles greater than Θ to the original direction is

$$\frac{1}{\pi}\cos^{-1}\left\{\frac{\cos\theta\cos\theta_c - \cos\Theta}{\sin\theta\sin\theta_c}\right\} \tag{10}$$

for arguments between -1 and +1; 0 or 1 outside the physical range.

4. Generation of Cerenkov light with EGS4

The EGS4 Monte Carlo generates individual tracks in segments. Continuous energy loss is applied between discrete interactions (primarily production of δ-rays at these energies) or over a preset maximum step length determined by the parameter ESTEPE. At the end of each step an angular deflection is generated, drawn from the Moliere distribution taken to second order only in 1/B. For very small steps, $b << 3$, every step becomes susceptible to the inadequacy of the Moliere form for $B \sim 1$, whereas in this GS treatment only a very small length at the beginning of the track is affected. For very large steps the EGS4 calculations can become inaccurate from inadequate sampling of quantities which depend on the electron energy. The limits of reasonable values of ESTEPE are material dependent, but the range 0.001 to 0.05 is generally applicable [6].

Within this range there remains a problem arising from the finite step size. If all Cerenkov photons from a given step are emitted at the Cerenkov angle relative to the direction of the straight track segment, the final pattern will be a series of cones. If the step size is doubled the number of cones is halved; the angular distribution of the Cerenkov light is thus sensitive to the step size [6,7]. This artifact is removed by linearly interpolating the local direction cosines of the track between successive steps [7].

5. Results.

Fig.1 shows the angular distribution of Cerenkov light for initial energies of 1, 5, 10 and 15 MeV. The GS calculations are compared with EGS4 results, using ESTEPE=0.01 and interpolation of the direction cosines [7]. (Results using ESTEPE=0.001 are not significantly different.) The agreement is almost perfect. Fig.2 shows the fraction of light emitted into the backward hemisphere ($\Theta > 90°$). The GS results lie systematically ~ 0.003 (in $\sim$0.1) below those generated with EGS4.

Such discrepancies as exist are of no conceivable practical importance and at this level of agreement it is anyone's guess which is closer to the truth. The GS method is completely different from that in EGS4 and the code completely independent of that in EGS4 and the Cerenkov interface. The conclusion is that the EGS4 code, with interpolation of the direction cosines in the Cerenkov interface, is validated and that EGS4 handles multiple scattering well, even at these low energies.

References

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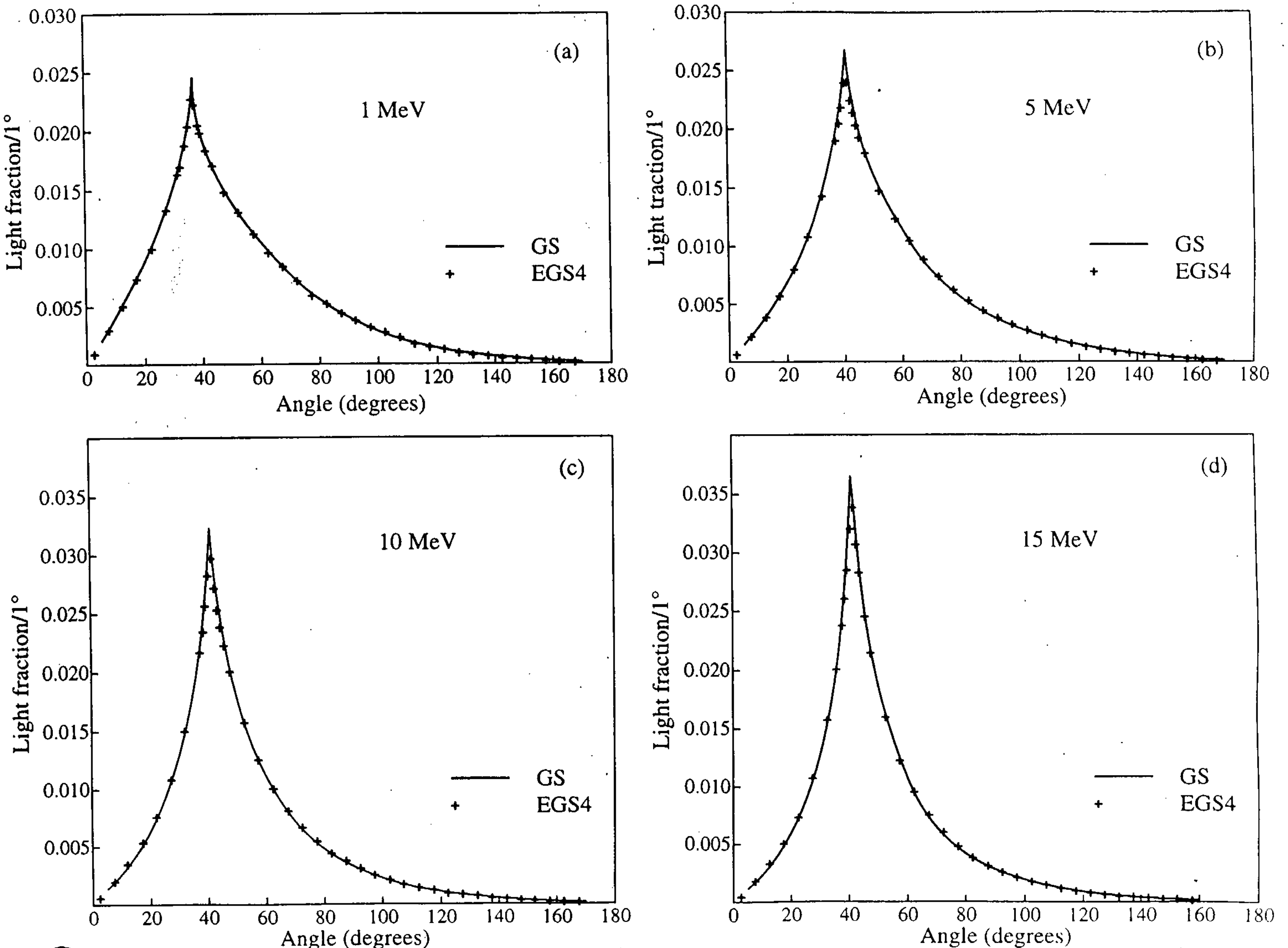
Figure captions

1. Angular distributions of the total Cerenkov light generated by electrons produced and stopping in water. The GS calculations are compared with results generated with EGS4. Light from δ rays is not included.

2. The fraction of Cerenkov light emitted into the backward hemisphere. The GS calculations are compared with EGS4 results and lie systematically very slightly below. Light from δ rays is not included.



(a)
1 MeV
(b)
5 MeV
(c)
10 MeV
(d)
15 MeV
Light fraction/1°
Angle (degrees)
GS
EGS4
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0
20
40
60
80
100
120
140
160
180

Fig. 1

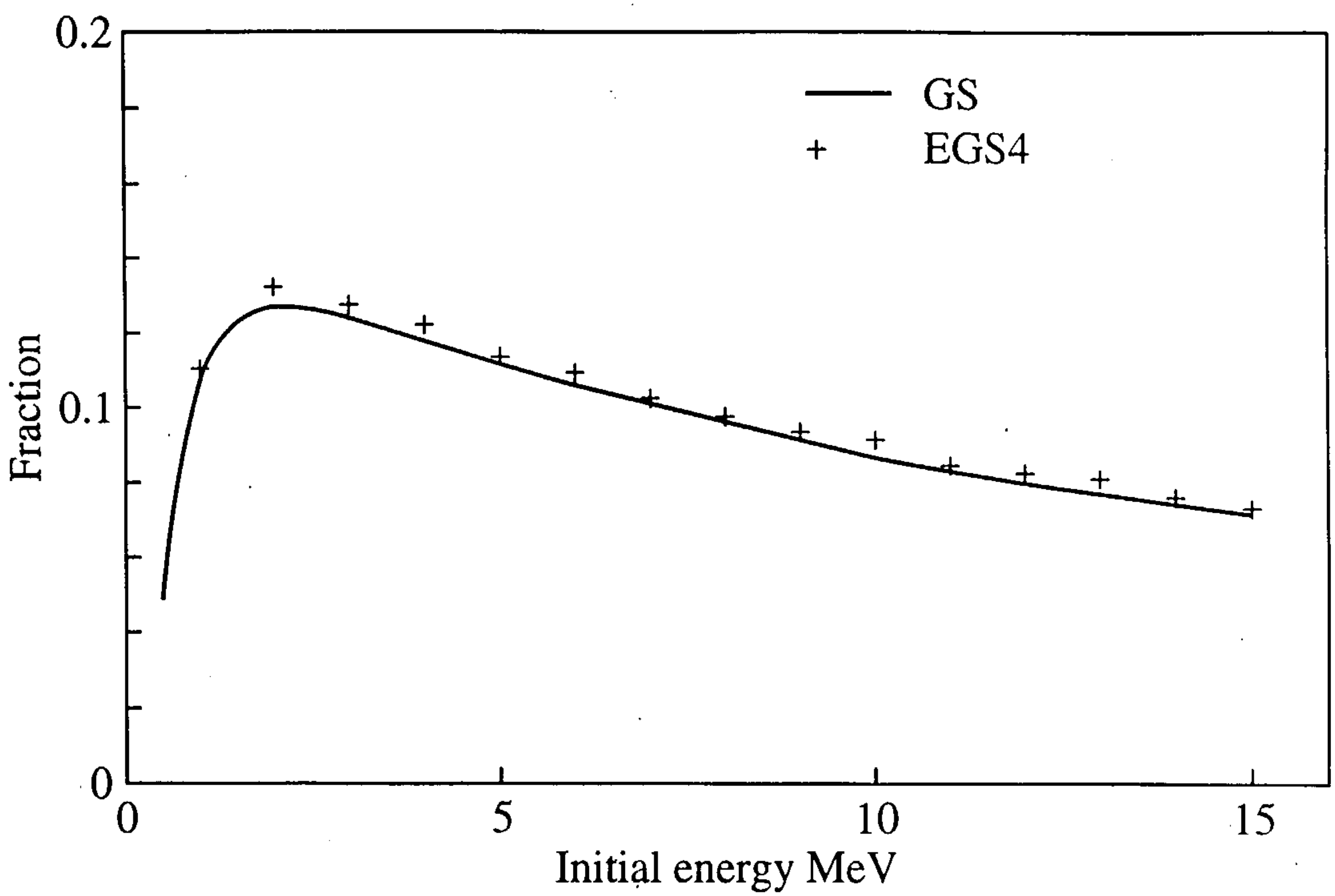


Fig. 2