HOWTO use the SNO Solar Neutrino Spectral Data

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1 Introduction

This document describes the data used in the SNO PRLs **Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory** [1] and **Measurement of Day and Night Neutrino Energy Spectra at SNO and Constraints on Neutrino Mixing Parameters** [2]. It describes how SNO produces its own contour plots, and alerts the reader to some pitfalls to be avoided. The SNO spectral data and supplementary information can be found on the SNO web site `http://www.sno.phy.queensu.ca`.

2 Warning About Using Results From Null Hypothesis Tests

The CC, ES, and NC fluxes presented in the neutral current PRL [1] , and the day-night asymmetries in the second PRL [2], are in general derived under the explicit assumption of an undistorted ^{8}B spectral shape (i.e., an energy-independent survival probability), except where explicitly stated otherwise.

The assumption of a ^{8}B spectrum is appropriate for testing the null hypothesis of neutrino oscillations. (We assume no oscillations, then show that this would imply a non-zero $\phi_{\mu\tau}$ at the 5.3σ level, thus refuting the null hypothesis.) However, using fluxes derived assuming an undistorted spectrum is not appropriate for calculating constraints on MSW mixing parameters, since MSW oscillations generally allow for spectral distortions. The spectral shape assumption is particularly important for distinguishing CC from NC events in the signal extraction, and if SNO were to repeat the signal extraction assuming some non-zero spectral distortion, in general we would get a different NC flux! In this sense, *the CC/NC ratio derived from the SNO data is model-dependent, and depends on the assumed shape of the CC energy spectrum.*

We caution against taking the CC, ES, and NC rates from the two PRLs, which were derived assuming no spectral distortions, and using them to derive constraints on models which allow for spectral distortions. Combining analyses in this manner is not internally consistent, and may lead to serious errors.

The one exception is the "unconstrained" NC flux quoted in the NC PRL:

$$\phi_{NC}^{SNO} = 6.42^{+1.57}_{-1.57}\,^{+0.55}_{-0.58} \times 10^6$$

This flux was derived without using any energy information, and in this respect can be used to test or constrain models which allow for spectral distortions. However, if the spectral shape is known or may be assumed (for example, when we are trying to determine how well a particular point in MSW parameter space describes the data), we will get a much tighter constraint from including energy information, and fitting that particular model (with spectral distortions) to the SNO total energy spectra. Energy spectra are extremely powerful for distinguishing CC from NC events, and constraining the CC/NC ratio.

SNO uses only the day and night energy spectra to produce contour plots, and not the rates given in the NC paper or the asymmetries in the day-night paper.

3 Correlations

We draw attention to the fact that our CC, ES, and NC fluxes are *statistically* correlated. The statistical correlation coefficient are given in the day-night PRL. They are the same for the total fluxes as they are for the day-night asymmetries on the fluxes.

Note as well that the ϕ_e and $\phi_{\mu\tau}$ fluxes are statistically correlated as well, with a correlation coefficient of -0.678.

Although we do not recommend deriving constraints on MSW parameters from the flux and asymmetry parameters directly (almost all of which were derived explicitly assuming an undistorted ^{8}B spectrum), any analyses which do make use of these fluxes (e.g. calculating the significance with which SNO rejects the null hypothesis of no neutrino oscillations) should take these correlations into account. The systematic uncertainties are also correlated. Table II of the NC PRL indicates anti-correlated systematic uncertainties between the CC and NC fluxes with a dagger. "Vertex accuracy" is the only positively correlated systematic of appreciable size.

4 Forward Fitting

SNO uses the "forward fitting" method to generate the MSW contours in Fig 4 of the DN PRL.

The basic idea is to convolve whatever theoretical model is being tested (e.g. a particular point in MSW parameter space) with SNO's response functions in order to generate predictions for the CC, ES, and NC energy spectra SNO should see. Adding in the energy spectra of low energy backgrounds (given on the SNO web site) will then yield a prediction for the total energy spectrum that SNO should see for all events. This prediction is then compared to the measured SNO energy spectrum (e.g. Fig 2c of the NC PRL or Fig 2a of the DN PRL), and a goodness of fit parameter calculated.

Because the total energy spectra contains the sum of CC, ES, and NC events, it encodes information about the CC/NC ratio in its shape. All of the flux information and shape information is contained in the spectrum itself.

In more detail, we do the following:

1. We start with a particular theoretical model for which we want to calculate a goodness of fit. For example, the model might be 2-ν oscillations with $\tan^2\theta = 0.4$ and $\Delta m^2 = 2 \times 10^{-5}$.

2. We calculate the electron neutrino survival probability as a function of energy, and the probability that the neutrino interacts as $\nu_{\mu\tau}$. (For active-only oscillations, these add to 1!)

3. We convolve the ^{8}B neutrino energy spectrum with the differential CC cross-section and the SNO energy response function to yield a prediction for the shape of the CC energy spectrum SNO should detect. We normalize the amplitude by SNO's livetime, the number of targets, etc. If the ^{8}B flux is allowed to float in the fit, an additional scale parameter is included on the normalization of the PDF.

4. We do the same thing for the ES reaction, remembering to include the contribution from $\nu_{\mu\tau}$ with the appropriate relative cross-sections.

5. Neutral current interactions generate a Gaussian PDF in energy, as described in the detector response section. In other words, the shape of the energy spectrum from neutron captures is independent of the neutrino energy. So we simply use the theoretical model to make a prediction for how many neutron capture events on deuterium SNO should have seen. Then we normalize the neutron energy PDF by this amount.

6. SNO's energy spectrum contains small numbers of events from radioactive backgrounds. These include background neutrons from sources such as photodisintegration, and Cherenkov tail events from $\beta - \gamma$ decays in the detector. The shapes and amplitudes of the background PDFs are given on the SNO web site, along with their uncertainties. The amplitudes are fixed by the SNO analysis, and so are not allowed to float as free parameters in the fit.

7. We sum the energy spectra for the CC, ES, NC, and background contributions. We then compare the resulting shape to the total energy spectrum from SNO. We evaluate a goodness of fit (e.g. χ^2 between the model spectrum and the data). We then repeat the procedure for other solutions or point in parameter space, and form $\Delta\chi^2$ contours as usual.

8. In this approach, the SNO energy spectrum gets assigned only statistical uncertainties. The systematic uncertainties in the SNO response functions (energy scale, neutron capture efficiency, etc) are treated as uncertainties on the model prediction for the energy spectrum. Similarly, uncertainties in the background amplitudes become systematic uncertainties on the model to which the SNO data gets compared. The systematic errors are of course correlated from bin-to-bin, but can be treated by the standard covariance matrix techniques.

Generalizations to day and night energy spectra are easily done.

The starting point for SNO's own MSW contour plots is always the total energy spectra, and never a CC, ES, or NC "flux", which are usually derived only for specific model assumptions.

5 SNO Spectral Data

The SNO spectral data presented in Figure 2 of [2] are day and night kinetic energy spectra for pure D_2O data. They contain contributions from the neutrino signals (CC, ES and NC) and backgrounds (Neutron and Low Energy). *These are not CC-only energy spectra!* They contain all three neutrino signals as well as neutron and low energy backgrounds.

The SNO spectral data for pure D_2O data set are tabulated in `SnoDayNightPrlSpectra.dat` and consist of separate spectra for "day" and "night". The data are binned in 16 0.5 MeV wide bins from kinetic energy, $T = 5$ MeV to 13 MeV. A 17th bin contains data from 13 MeV to 20 MeV. Day is defined a zenith angles less than $90°$, night as zenith angles greater than $90°$. SNO was not 100% live during the data taking period. The day livetime was 128.5 days, the night livetime was 177.9 days. The distribution of the solar elevation sampled by the experiment is tabulates in $\cos\theta_z$, the cosine of the zenith angle and is listed in `SnoCosZenith.dat`.

6 SNO Backgrounds

The spectral data in `SnoDayNightPrlSpectra.dat` contain background events that have been divided into two categories: *Neutron Backgrounds* and *Low Energy Backgrounds*. The number of events of each type and their uncertainties are tabulated in `SnoBackgroundTables.dat`

The two classes of backgrounds correspond to the "total neutron background" and "total Cherenkov background" entries of Table I of the NC PRL.

Note that for each background, the errors for the different day and night energy bins are completely correlated. Put another way, it is really the *amplitude* of the PDF, and not its shape, which is uncertain. All the day and night energy bins will move together as the amplitude is varied up or down.

For the low energy background (aka "Cherenkov tail" background), the asymmetric error bars in Table I of the NC PRL have been symmetrized by taking their average.

The neutron backgrounds and the Cherenkov tail backgrounds to first approximation are independent of each other, and can be treated independently. (The uncertainties on the amplitudes are mostly independent, and so the bin-to-bin correlations between the two PDFs are small.)

7 SNO Energy Response

The SNO detector response for electrons was characterized as a Gaussian function:

$$R(T_e, T) = \frac{1}{\sqrt{2\pi}\sigma_T} \exp\left[\frac{-(T_e - T)^2}{2\sigma_T^2}\right] \tag{1}$$

with resolution:

$$\sigma_T = -0.0684 + 0.331\sqrt{T_e} + 0.0425 T_e \text{ MeV} \tag{2}$$

where T_e is the true electron kinetic energy in MeV, and T is the kinetic energy SNO assigns to the event. The response to neutrons is similar except that the energy deposit is monoenergetic:

$$R_{NC}(T) = \frac{1}{\sqrt{2\pi}\sigma_{NC}} \exp\left[\frac{-(T_{NC} - T)^2}{2\sigma_{NC}^2}\right] \tag{3}$$

where $T_{NC} = 5.08 MeV$ and $\sigma_{NC} = 1.11 MeV$.

8 Neutron Capture Efficiency

As detailed in the NC paper, the capture efficiency for NC events in pure D_2O is $29.9\% \pm 1.1\%$. That is the probability that a neutron generated at a random location inside the 600 cm radius acrylic vessel will capture on a deuteron. By the time the neutron captures, it will have thermalized, and have lost any information about its original direction or energy. All detected NC events in SNO's pure D_2O phase look like 6.25 MeV gamma rays.

Not all of the captured neutrons will be inside SNO's fiducial volume and above SNO's energy threshold. For neutrons generated inside the acrylic vessel, $14.4\% \pm 0.53\%$ will be detected inside 550 cm and above $T > 5$ MeV.

So, to determine how many neutrons SNO should detect from the NC interaction, we must first calculate how many neutrons are produced inside the *entire acrylic vessel (600 cm radius)*, then multiply by 14.4%. We must then apply a correction to account for cut losses (described below).

It's important to remember that the 14.4% number refers to the detection efficiency for events produced throughout the AV. Because the neutrons migrate a while before capturing, SNO has sensitivity to neutrons produced throughout the D_2O, even outside our fiducial volume cut of 550 cm. (The story is a little different for CC and ES events. These events produce Cherenkov light right at the point of the neutrino interaction. Because of SNO's finite vertex resolution, they may reconstruct a little bit away from the the interaction point. However, with a fiducial volume cut of 550 cm, as many CC and ES events get pushed outside the fiducial volume by vertex resolution as get pushed inside. So the fraction of CC and ES events produced inside the AV which reconstruct inside our fiducial volume is just $(550/600)^3$.)

For the purposes of constructing energy spectra, it is more useful to break the total detection probability of 14.4% into an energy component and a radial component. This is because systematic errors in energy scale or radial reconstruction will affect the 14.4% number. Doing this, we find that:

- $27.01\% \pm 0.99\%$ of NC neutrons capture on deuterons inside 550 cm, making events of some energy
- 53.2% of all neutrons have energies above $T > 5$ MeV (see Section 7).

The product of these two quantities is in fact 14.4%. But if an error was made in energy scale (affecting what fraction of the neutron PDF is above $T > 5$ MeV, for example), we can recalculate a new neutron detection efficiency by reevaluating what fraction of the neutron energy PDF is above the threshold, then multiplying by the "radial" part 27.01%.

9 Signal Loss from Cuts

SNO applies a number of event cuts to remove instrumental contamination, etc. These cuts remove a small fraction of the signal events.

The signal loss for each signal reaction is shown in Table 1.

Reaction	Signal loss from cuts
CC	$(1.43\pm^{+0.39}_{0.21})\%$
ES	$(1.46\pm^{+0.40}_{0.21})\%$
Neutrons	$(2.28\pm^{+0.41}_{0.23})\%$

Table 1: Cut sacrifice for the CC, ES, and NC reactions

Background amplitudes do not need to be corrected for cut loss—the numbers SNO provides for these are estimates of the number of detected background events, and so already include cut loss.

The day and night cut loss values are identical, and their uncertainties are basically uncorrelated.

We multiply the model prediction for each signal by one minus the loss fraction to get the prediction for how many events of each type should actually be in the SNO data set, after cuts.

10 Target Volume

The acrylic vessel contains 1.003×10^6 kg of heavy water. This works out to be 6.027×10^{31} deuterons. There is also a small amount of D_2O in the "neck" of the acrylic vessel, which gives a negligible contribution to the CC, ES, and NC signals. The acrylic vessel is 600 cm in radius.

11 Location of SNO

SNO is located at $46°28'30''$ N latitude and $81°12'04''$ W longitude at a depth of 1730 m below sea level (2039 m below the surface) [3].

12 Systematic Uncertainties in the SNO Spectra

In addition to the statistical uncertainties for each bin in the SNO spectra, there are systematic uncertainties on the detector response functions. In the forward fitting technique, these result in systematic uncertainties on the model prediction for the total energy spectrum.

Each systematic is generally an uncertainty in a response function parameter (e.g. energy scale, acceptance of the fiducial volume cut, etc.) These uncertainties will generally affect all bins of the energy spectrum in a correlated way. SNO constructs $N \times N$ covariance matrices for the systematics, which are then added to the statistical uncertainties on the spectral data (a diagonal matrix) to get a total uncertainty matrix that is used to form the SNO χ^2.

Here we discuss the dominant systematics:

12.1 Energy Scale

By far the most important systematic uncertainty for SNO is the accuracy with which we have determined the absolute energy scale in the detector. The *total* energy ($T + 0.51$ MeV) of each event is known to $\pm 1.21\%$. The x-axis of our total energy spectrum could be scaled by this amount.

Equation 1 gives the SNO energy response. Ideally, the energy T we assign to each event would be equal on average to the true event energy T_e. If SNO has miscalibrated its energy scale by a small amount, then an electron with true total energy $E = T + 0.511$ MeV will actually get assigned on average a total energy of $E_e = (1 + \alpha)E$. Here, $\alpha = 0 \pm 0.0121$ is the energy scale uncertainty. Please note that the energy scale uncertainty is an uncertainty on the *total* electron energy.

A shift in energy scale will also change the acceptance of the energy threshold cut, i.e. the number of signal events above the energy threshold.

12.2 Diurnal Energy Scale

As discussed in Section 12.1, the absolute energy calibration of the SNO detector is uncertain within $\pm 1.21\%$. However, the difference between the day and night energy scales is known to much greater precision. The diurnal energy scale difference between day and night energy scales is limited to be less than $\pm 0.3\%$.

12.3 Vertex Accuracy

On average, events should reconstruct at the location where the Cherenking electron originated. Due to inaccuracies in event reconstruction, the radial scale for event reconstruction is uncertain by $\pm 1\%$. In other words, the actual fiducial volume of the analysis might actually be 555.5 cm of 544.5 cm, rather than the nominal value of 550 cm. This uncertainty variously gets referred to as "vertex accuracy", or "vertex shift", or "radial scale error".

This uncertainty affects how many events reconstruct inside the fiducial volume. For CC and ES events, a 1% change in radial scale will shift 3% of the total number events across the fiducial volume cut, by simple volume scaling. The NC radial PDF is strongly peaked to the center, however—the change in the number of NC events inside the fiducial volume per 1% shift in radius is 1.45%. (In other words, the fraction of NC events reconstructing inside 550 cm is actually $27.01\% \times (1 \pm 0.0145)$—see Section 8).

12.4 Diurnal Vertex Accuracy

As detaled in Section 12.3, the radial scale for reconstruction is uncertain to $\pm 1\%$. (In other words, the true location of our fiducial volume cut is uncertain at that level.) The difference between the day and night radial scales is limited to be less than $\pm 0.2\%$.

12.5 Neutron Capture Efficiency

The neutron capture efficiency and uncertainty is given in Section 8. The neutron capture efficiency has a relative uncertainty of $1.1/29.9 = 3.7\%$. The number of detected neutrons has an uncertainty of this magnitude.

This uncertainty is separate from the uncertainty on what fraction of the neutrons will be above the energy threshold (which is determined by the energy scale uncertainty), and the uncertainty on what fraction of the neutrons are detected inside 550 cm due to radial scale uncertainty. (See Sections 12.1 and 12.3).

Note that a change in neutron capture efficiency will also scale the number of background neutrons (see Section 6). The uncertainties on the background PDFs do not include this component.

12.6 Energy Resolution

The energy resolution of the SNO detector is detailed in Section 7. The actual energy resolution of the detector is uncertain, and the size of that uncertainty increases with the energy of the event.

The uncertainty on the energy resolution at any energy has been parameterized by:

$$\frac{\Delta\sigma_T}{\sigma_T} = 0.045 + 0.00401 \times (T_e - 4.98) \tag{4}$$

For $T < 4.98$ MeV, $\Delta\sigma_T/\sigma_T = 0.045$. The width of the NC energy PDF is also uncertain by $\pm 4.5\%$. (The NC PDF width uncertainty and the electron energy resolution uncertainty are totally correlated.)

12.7 Energy non-linearity

SNO determines the energy scale in the detector primarily using an ^{16}N source of 6.1 MeV gamma rays. Having fixed the energy scale at this point, there may be residual uncertainty on how the energy scale extrapolates to higher energies.

We have parameterized the error in event energy from such non-linearities according to the following formula:

$$\delta T = \alpha \times 19.1 \times \frac{T - 4.98}{13.61} \tag{5}$$

where α is limited to ± 0.0023, and T is the energy assigned to the event. The error from this component is always zero at the ^{16}N energy of 4.98 MeV, then increases linearly with energy.

12.8 Vertex Resolution

Vertex resolution is a very small component of SNO's overall uncertainty, and can probably be neglected for most purposes. For completeness, we note that the vertex resolution *for each coordinate* is approximately a Gaussian with a width of $\sigma_x =\sim 18 \pm 2$ cm, with small non-Gaussian tails. (This is the distribution of the difference between the real event's x-coordinate and the fitted x-coordinate. The resolution is the same for all three coordinates.)

Because SNO's fiducial volume cut is nearly 3σ away from the edge of the acrylic vessel, a change in vertex resolution has a negligible impact on the number of events inside the fiducial volume. Only the sloped NC PDF shows any effect—a ± 2 cm change in the vertex resolution changes the number of accepted neutron events by 0.1%.

12.9 Cut acceptance

Details and uncertainties on cut acceptance are given in Section 9.

12.10 Angular resolution

SNO's MSW contour plots do not include any angular information, although it is used when doing signal extraction for null hypothesis tests.

12.11 Background Uncertainties

Uncertainties on the amplitudes of neutron and Cherenkov backgrounds are given in Section 6. SNO treats uncertainties in background PDFs as systematic uncertainties on the predicted energy spectrum that gets compared to SNO's measured energy spectrum.

12.12 A word on other day/night systematics

We have treated here SNO's dominant systematics, including all of the systematics in Table II of the NC PRL, and the most important day-night differential systematics (diurnal energy scale variations, diurnal vertex shift, and sacrifice). There are a number of smaller differential systematics between day and night, particularly directional systematics that only have an appreciable effect for the ES flux. Because our day/night results are dominated by statistics and not systematics, we have neglected these smaller systematics, and have explicitly included only the largest day-night systematics which affect the CC and NC flux asymmetries.

13 Contact Persons

For more information about how SNO produces MSW contours from its data, you may contact the following individuals, who will forward your questions to the appropriate parties:

- Mark Chen (mchen@owl.phy.queensu.ca)
- Fraser Duncan (fduncan@surf.sno.laurentian.ca)

References

[1] *Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory* , Q.R. Ahmad *et al*, `nucl-ex/0204008`

[2] *Measurement of Day and Night Neutrino Energy Spectra at SNO and Constraints on Neutrino Mixing Parameters*, Q.R. Ahmad *et al*, `nucl-ex/0204009`

[3] *The Sudbury Neutrino Observatory*, The SNO collaboration, Nucl. Instr. and Meth. A449, 172 (220).