ACCIDENTAL TRIGGER RATES

W. Frati
University of Pennsylvania
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I. Introduction

The trigger threshhold for accepting an event is set by requiring some number of PMT's, n_t, to fire within some resolving time. Three types of accidental triggers are considered: 1) n_t PMT's randomly occurring within the resolving time; 2) A subset of n_t arising from a low energy event and the remainder coming from randomly firing PMT's; 3) Two low energy events occuring within the resolving time adding up to n_t.

I. n_t RANDOMLY FIRING PMT'S

The accidental rate, A, due to n_t PMT's firing a discriminator EACH of width τ and overlapping in time is given by

$$A = n_t \tau^{-1} (R\tau)^{n_t} (1 - R\tau)^{N-n_t} \binom{N}{n_t}$$

where

R: PMT singles rate

N: Total number of PMT's

$\binom{N}{n_t}$: Binomial coefficient

Using the fact that $N \gg n_t$, the following approximations can be made:

$$(1 - R\tau)^{N-n_t} \approx (1 - R\tau)^N \approx e^{-NR\tau}$$

$$\binom{N}{n_t} = \frac{N!}{(N - n_t)!\, n_t!} \approx \frac{N^{n_t}}{n_t!}$$

The resulting equation that is used for the calculations in this report is:

$$A = \frac{\tau^{-1} k^{n_t} e^{-k}}{(n_t - 1)!} \qquad : k = NR\tau$$

For values of A = 0.01, 0.10, 1.0, 10., and 100. Hz , the value of $NR\tau$ is calculated for various values of n_t, for τ= 80 and 100 ns. The results are tabulated in Table I, where it is noted that $NR\tau$ doesn't change much in going from τ = 80 ns to 100 ns. Figure 1 is the plot of $NR\tau$ as a function of n_t for τ= 100ns, with the lines drawn through the points of constant A.

So for example, if we require 20 PMT's to trigger the electronics where each PMT discriminator puts out a 100 ns pulse, and want the accidental rate to be less than 1 Hz, then from Table I or Fig 1, $NR\tau$= 3.88. For 5600 PMT's, the PMT singles rate must be less than 6921 Hz, which calculates to 15 Hz/cm^2 for the 460 cm^2 Burle tube.

It is to be noted that the above calculation is a for a differential spectrum and not an integral one, ie the trigger rate when the discriminator is set for n_t is the sum of all the rates from n_t on up. However when the spectrum is falling steeply, the differential number is a reasonable approximation. Thus for the numbers used above, the differential rate for 21 PMT's firing is 1/5 that of the differential rate for 20 PMT's firing, and the actual trigger rate with the trigger threshold set at 20 PMT's is $\approx$ 25% higher than the 1 Hz calculated above.

II. LOW ENERGY EVENT PLUS PMT RANDOMS

Fig 2 shows the event rate from the Th and U background as a function of the number of firing PMT's. This is from the Queens MC using 8856 generic 8" PMT's with 2.5 ns FWHM and 56 degree reflectors. I have approximated this by the exponential shown in Fig 2, which is given by

$$A_e = B_0 e^{-\alpha n_1} \text{ Hz}$$

where α = 0.33 , B_0 = 300, and n_1 is the number of PMT's fired by the low energy event within a 100 ns window.

Again note that this is a differential distribution and when summed over all values greater than or equal to n_1 one gets $\sum_{m=n_1}^{\infty} A_e(m) \approx 3.5 A_e(n_1)$

The differential rate for n_2 randomly firing PMT's is

$$A_r = \frac{\tau^{-1} k^{n_2} e^{-k}}{(n_2 - 1)!} \qquad : k = NR\tau$$

The two fold accidental rate of A_r and A_e is given by

$$A = \sum_{n_1} \sum_{n_2} A_r A_e \tau \delta(n_t - n_1 - n_2)$$

I have used τ rather than 2τ, since I assume the pulse width from the accidental coincidences will be small compared to that of the low energy event, which I take to be the full resolving time.

$$A = \sum_{n_2=1}^{n_t} B_0 e^{-\alpha(n_t - n_2)} (k)^{n_2} e^{-k} / (n_2 - 1)!$$

$$A/A_e = e^{-k} \sum [ke^{-\alpha}]^{n_2} / (n_2 - 1)!$$

$$A/A_e = e^{-k} \beta e^{\beta} \qquad : \beta = ke^{\alpha} \qquad : n_t > 7$$

Following up on the numerical example in section II consider

N: 5600 PMT's

R: 5000 Hz

τ: 100 ns

α: 0.33

then

$k = NR\tau = 2.8$

$\beta = ke^{\alpha} = 3.91$

$A/A_e = 11.9$

This number drops to 2.3 with a PMT noise rate of 2000 Hz.

These types of triggers are potential problems only at the trigger rate level. We should have no trouble in weeding most of them out with timing cuts and reconstruction.

The numerical values of the parameters for the measured KamiokandeII trigger rate are

$\alpha = 0.59$, $B_0 = 10^5 Hz$.

III. TWO RANDOM LOW ENERGY EVENTS

The trigger rate from two random low energy events is given by the convolution of the single low energy event spectrum shown in Fig 2.

$$A_2(n_t) = 2\tau \sum_{n_1=1}^{n_t-1} \sum_{n_2=1}^{n_t-1} A_e(n_1)A_e(n_2)\delta(n_t - n_1 - n_2)$$

$$A_2(n_t) = 2\tau(n_t - 1){B_0}^2 e^{-\alpha n_t}$$

Thus the ratio of the accidental rate to the background rate is

$$A_2(n_t)/A_e(n_t) = 2\tau(n_t - 1)B_0$$

Thus for NHIT = 20 the accidentals are 0.1% of the background rate.

Table I

NRT VS ACCIDENTAL RATE RES TIME = 80.00 NS

NTRG	.01	0.1	1.0	10.0	100. ← Accid Rate H2
14	1.224	1.468	1.768	2.140	2.609
16	1.719	2.024	2.391	2.840	3.395
18	2.281	2.645	3.079	3.602	4.242
20	2.900	3.323	3.923	4.420	5.141
22	3.571	4.053	4.617	5.285	6.085
24	4.290	4.829	5.456	6.193	7.070
26	5.051	5.646	6.335	7.139	8.091
28	5.850	6.501	7.249	8.119	9.143
30	6.685	7.390	8.197	9.131	10.225
32	7.552	8.310	9.174	10.171	11.334
34	8.449	9.259	10.179	11.237	12.467
36	9.373	10.234	11.210	12.327	13.623
38	10.323	11.234	12.264	13.439	14.799

NRT VS ACCIDENTAL RATE RES TIME = 100.00 NS

NTRG	.01	0.1	1.0	10.0	100. ← Accid Rate H2
14	1.246	1.495	1.801	2.181	2.661
16	1.747	2.056	2.431	2.889	3.456
18	2.313	2.683	3.125	3.659	4.312
20	2.938	3.368	3.876	4.483	5.219
22	3.615	4.104	4.677	5.356	6.171
24	4.339	4.886	5.522	6.271	7.164
26	5.105	5.709	6.407	7.224	8.192
28	5.910	6.569	7.328	8.211	9.253
30	6.749	7.463	8.281	9.229	10.342
32	7.621	8.389	9.265	10.275	11.458
34	8.523	9.343	10.275	11.348	12.598
36	9.452	10.323	11.312	12.444	13.760
38	10.407	11.329	12.371	13.562	14.943

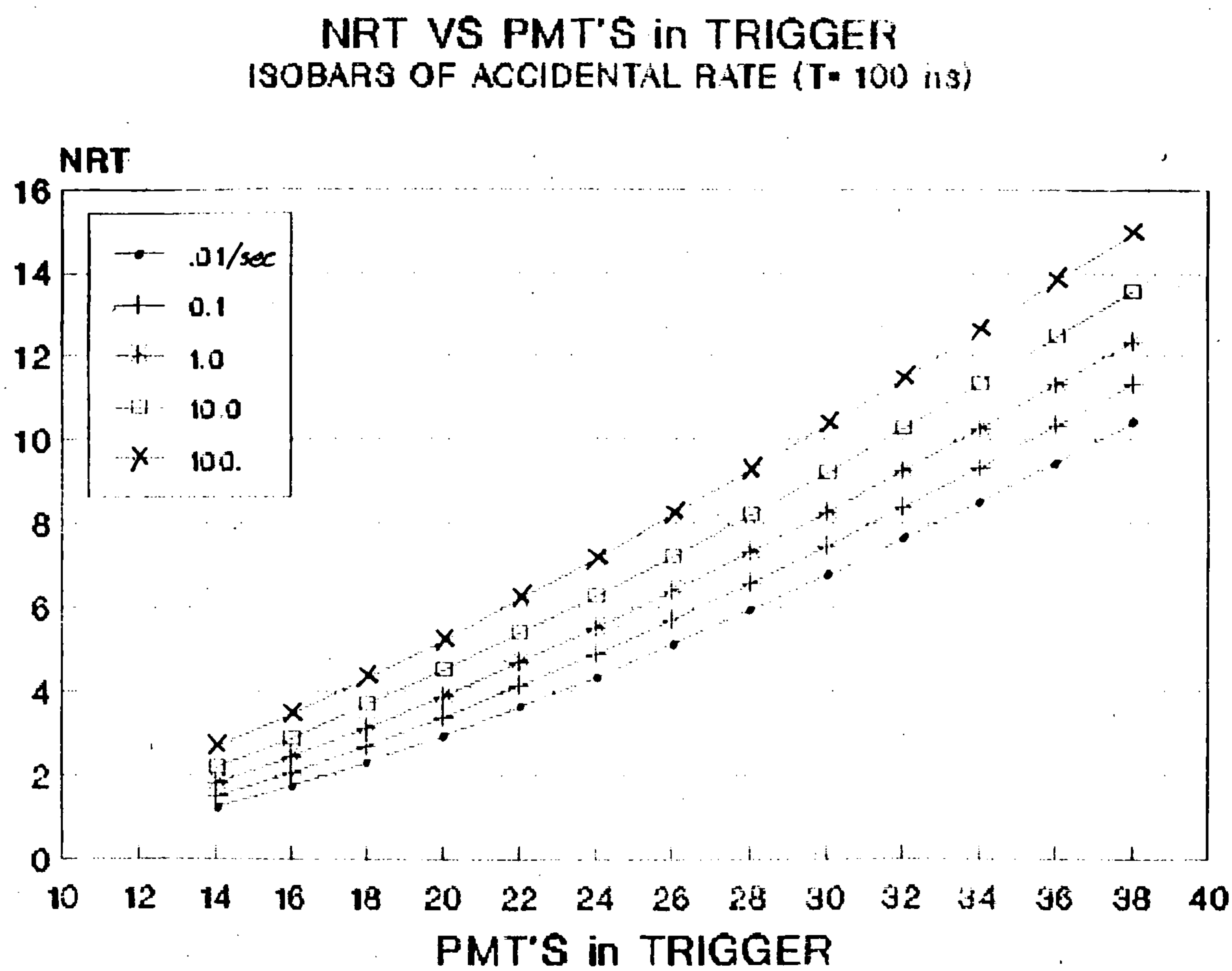
NRT VS PMT'S in TRIGGER
ISOBARS OF ACCIDENTAL RATE (T= 100 ns)
NRT
.01/sec
0.1
1.0
10.0
100.
0
2
4
6
8
10
12
14
16
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
PMT'S in TRIGGER

Fig 1

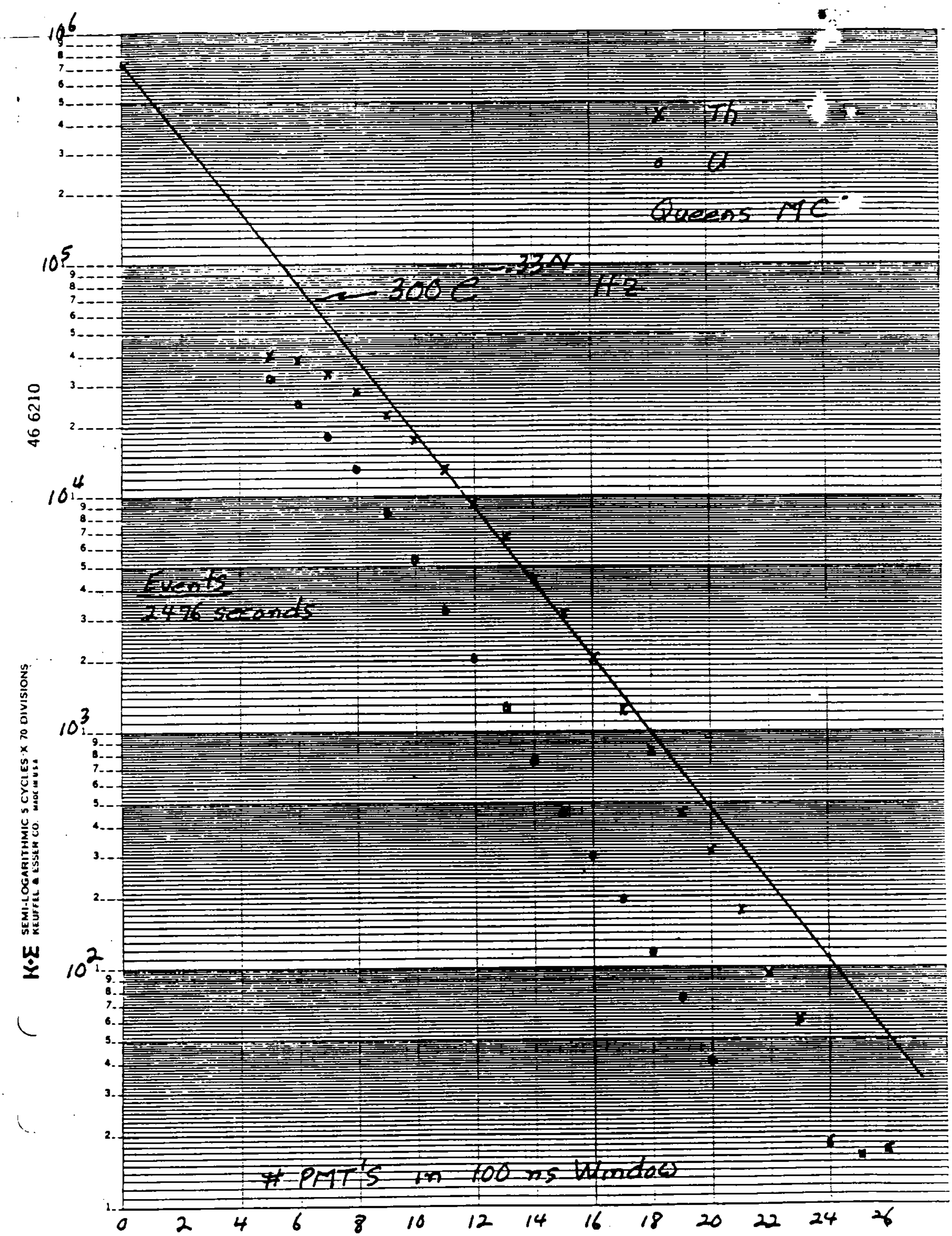
x Th
o U
Queens MC
300 C
-33N
H2
Events
2476 seconds
PMT'S in 100 ns Window
46 6210
K·E SEMI-LOGARITHMIC 5 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

Fig 2