The impedance for Water Flow through the Photomultiplier Sphere

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1. Introduction

It is extremely desirable to achieve a high water flow pattern through the photomultiplier sphere which is everywhere radially outward to avoid the risk of contamination, radioactive and optical, from the relatively dirty components outside the sphere. To do so it is necessary to obtain an over pressure within the sphere which is large enough to prevent the turbulent convection current outside the sphere from passing through the sphere.

In the ASC Report no.99-1022-1 it was estimated (with some uncertainty) from the preliminary turbulent convection calculations that a sufficient overpressure would be obtained with the recirculation flow of 130 m/s of the photomultiplier sphere blocked the flow to about 92.2%, equivalent to an orifice of \( \sim 1 \text{ cm}^2 \) per photomultiplier (see Section 4.6 and Appendix B of the ASC Report). Subsequently the target for the flow impedance of the sphere was set as an orifice of 0.4 cm\(^2\) per photomultiplier.

The pressure drop \( \Delta p \) through an orifice, of diameter \( d \) thickness of material, is related to the velocity of flow \( V \) in the orifice by the Bernoulli equation

\[
\Delta p = \frac{1}{2} \rho V^2,
\]

and is approximately independent of the viscosity. For the recirculation flow of \( Q_c \sim 2.25 \times 10^{-2} \text{ m}^3/\text{s} \) and an orifice of 0.4 cm\(^2\), both per photomultiplier, \( V = 5.6 \times 10^4 \text{ cm}^3/\text{s} \) and \( \Delta p = 0.0157 \text{ Pa} \). This pressure head \( \Delta p \) and the recirculation flow \( Q_c \) are taken as the design parameters for the water tightness of the photomultiplier sphere. For impedances it is convenient to work in units of \( Z_0 = \Delta p/Q_c \).

The flow apertures which occur between the photomultipliers, concentri,

\[
\frac{Q}{\pi R^2 \Delta p / \rho g L} = 4.72 \frac{h^2}{L} \text{ m}^3 \text{ s}^{-1}
\]

\[ \tag{I} \]

for \( \Delta p = 0.0157 \text{ Pa} \) and \( \eta = 1.307 \times 10^{-3} \text{ Pa.s} \) for water at 10°C.

(ii) Long slit of gap \( x \), length \( L \) in the flow direction, and width \( w \),

\[
Q = \Delta p w x^2 / 12 u L
\]

\[
= 1.001 w x^2 / L \text{ m}^3 \text{ s}^{-1}
\]

\[ \tag{II} \]
(iii) Annular pipe of radii $R_1$ and $R_2 = R_1 + \Delta R$, and length $L$,

\[ Q = \pi \Delta R (R_2^2 - R_1^2) \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \frac{p_2 - p_1}{\rho g L} \]

\[ = \pi \Delta p R_1 \Delta R \rho g L \] for $\Delta R < R_1$  \hspace{1cm} (III)

\[ = 0.26 R_1 \Delta R^3 \rho g L \, \text{m}^3 \text{s}^{-1} \]

2. Calculation of Flow Rates and Impedances

Fig. 1 shows the essential assembly of the photomultiplier, concentrator, and hexagon. It is of interest to consider the water flows and impedances without the use of the silicon rubber indicated in the figure, or concentration gasket.

(a) Air bleed holes

These are sets of 6 holes of 2 mm diameter at the vertices of the hexagon and also in the concentrator disk (not shown). The thickness of the material is about 0.1 m. Using equation (II) gives:

\[ R = 1.5 \times 10^{-3} \text{m} \]

\[ Q_1 = 2.59 \times 10^{-2} \text{Q}_{\text{e}} \text{ per 0 holes} \]

and

\[ Z_1 = 28 \times Z_0 \]

\[ R = 1.5 \times 10^{-3} \text{m} \]

\[ L = 5 \times 10^{-2} \text{m} \]

\[ Q_2 = 0.125 Q_1 \text{ per 6 holes} \]

\[ Z_2 = 7.84 Z_1 \text{ per 0 holes} \]

(b) Front rim of concentrator/hexagon

The concentrator is free to move in the radial direction relative to the PMT-baffle and is held in place by the spring loading on the photomultiplier which is indicated schematically in Fig. 1. The act had on the concentrator, allowing for buoyancy, might be chosen to fall in the range 2 to 10 kg, depending on the orientation of the hexagon. This is a modest load and the magnitude of the effective gap, $x$ in equation (IV), between the rim of the concentrator and the front of the hexagon will depend on the fitness of the injection molded plastic components and the possible use of washers at the vertices of the hexagon to stiffen the front. For the purpose of making an estimate it will be assumed $x = 0.5 \text{mm}, L = 3 \text{mm},$ and $n = 600$ in equation (IV) giving

\[ Q_x = 0.150 Q_2 \]

and

\[ Z_x = 0.28 Z_2 \]

with an uncertainty of something like a factor of 10.
(c) Concentrator/PMT Baffle

This is a cylindrical sliding joint for which the flow is given by equation (11) with \( \Delta F = 2.26 \text{mm} \) (maximum), \( R_i = 111 \text{mm} \), and \( L = 158 \text{mm} \) (nominal). This gives

\[
Q_{cp} = 3.2 \times 10^{-3} Q_0
\]

and \( Z_{cp} = 309 Z_0 \).

(d) PMT Baffle/Hexagon

The perimeter of the hexagonal shape of the PMT Baffle has a length \( u = 600 \text{mm} \) and a thickness \( L = 6 \text{mm} \). The effective gap will probably be determined by the distortion of the hexagons under load but \( x = 0.5 \text{mm} \) might be a plausible guess. Then from equation (11)

\[
Q_{BII} = 0.110 Q_0
\]

\( Z_{BII} = 0.05 Z_0 \).

(e) Hexagon/Hexagon

The perimeter of the hexagon is \( u = 1034 \text{mm} \) and the length \( L = 158 \text{mm} \). For an effective gap \( x = 2 \text{mm} \) between adjacent hexagons, the half flow which can be assigned to one photomultiplier unit is given by equation (11) as

\[
Q_{III} = 3.04 \times 10^{-3} Q_0
\]

\( Z_{III} = 22.9 Z_0 \).

(f) Concentrator/Photomultiplier

Where the photomultiplier is in contact with the concentrator, the photomultiplier surface has a convex radius of 0.88mm and a concave radius of 0.44mm. The difference of maximum and minimum diameter of the photomultiplier at the concentrator location has been measured at Queen’s U. and LBL for 60 photomultipliers with similar results: Queen’s 0.004 ± 0.003mm, LBL 0.005 ± 0.002mm (mean and standard deviation). The measuring surface of the concentrator is a machined cone of half angle 20°, the surface of which is tangential to the photomultiplier and has a length of 48mm. Treating the concentrator/photomultiplier as a circle overmolding an ellipse and averaging over the geometry and the statistic distribution gives an effective gap perpendicular to the photomultiplier surface of \( \sqrt{CF}^2 = 0.227 \text{mm} \) in equation (11), with \( r = 4 \text{mm} \) and \( W = 31 \text{mm} \).

\[
Q_{CP} = 1.42 \times 10^{-3} Q_0
\]

\( Z_{CP} = 1.34 \times 10^4 Z_0 \).

It is clear that the eccentricity of the unsmoothed photomultiplier is a negligible consideration. The actual gap could become larger because of distortion of the plastic component on the relaxation of the molding streams after machining, but the geometry and loading are very favourable and it seems unlikely that
Q would exceed one per cent of Qa, and we shall adopt \( Q_{CP} \leq 10^{-5} Qa \).
\( Z_{CP} \geq 100 Zs \).

3. The Net Impedance to Flow

From Fig.1 total flow impedance per photomultiplier amounts to

\[
Z_F = \left( \left( Z_1^{-1} + Z_2^{-1} \right) \cdot \left( Z_{CB}^{-1} + Z_{CH}^{-1} \right) \right)^{-1}
\]

with

\[
Z_1 = 39.7 Z_0
\]

\[ Z_2 = 7.84 Z_0 \]

\[ Z_F = 0.28 Z_0 \]

with an uncertainty of a factor of \( \pm 10 \).

\[ Z_{CD} = 300 Z_0 \]

\[ Z_{BH} = 9.6 Z_0 \]

\[ Z_{HH} = 32.9 Z_0 \]

\[ Z_{CP} > 100 Z_0 \]

With the estimates as given \( Z_F = 0.34 Z_0 \). If \( Z_F \) is small then \( Z_F = 5.59 Z_0 \).

In both cases \( Z_{BH} \) is the controlling impedance and if this were made large, by reducing \( z \) or increasing \( Z \) or sealing with silicone rubber, \( Z_F \) could be made as large as 14.5 \( Z_0 \) independent of \( Z_F \).

4. Conclusions

(i) The impedance for water flow through and between the hexagonal could reasonably be made an order of magnitude higher than the effective impedance of a 40mm² orifice per photomultiplier.

(ii) Whether a correspondingly high impedance can be achieved with the seal between the hexagonal panels is something which LBL will need to consider.

(iii) If an overall net impedance of 10 \( Z_0 \) could be obtained this would raise the pressure head to 0.157 \( Z_0 \) and give a much more comfortable safety factor with respect to convection currents flowing inward through the photomultiplier sphere.

(iv) There is no case at \( Z_0 \) for sealing with silicone rubber or elastomer, the concentrator to the photomultiplier or the PMT Baffle to the concentrator.

(v) There is a case for improving the seal between the PMT Baffle and the hexagon, which, in a manner which suits with LBL.

(vi) It will not be possible to predict the impedance \( Z_F \) with any confidence until injection moulded prototypes are available.

(vii) If \( Z_F \) is small \( Z_F \) there will be very little flow through the concentrator dish. However even if \( Z_F \) is large it is still likely that the turbulent convection currents within the photomultiplier sphere will mix any heating products from the concentrator and photomultiplier into the main body of water.
(viii) It is quite impractical to simulate in the laboratory the flow conditions under the minute pressure head \(\Delta p\). On the other hand it is easy to measure \(\Delta p\) of significant size, typically several tenths of a mm.

(ix) There is a powerful case for mounting directional flow sensors on the PMT balls inside the hexagons. How else will we know whether or not there is radial inwards flow?

(a) The impedance of the bleed holes for air is a factor of \(1.4 \times 10^{-2}\) less than for water.