1. Introduction

It is proposed to alternate the operation of the detector with and without NaCl in solution at the level of 0.25%, with a periodicity of some months. During the processes of salination and desalination, and more generally and importantly during any period of homogenization of the H_2O saline solution, it may be difficult to interpret the data recorded, and for this reason it is desirable to minimize the changes of these times.

The main purpose of the note is to enhance the numerical values of the various parameters which enter the problem and the few numbers of fluid dynamics which can be applied. The numbers relate in fact as it is difficult to find a complete set for D_2O, and unlikely that D_2O will be very different, apart from the numerical value of the density. The formulas have been taken from Buckingham, Fluid Mechanics, Progress reports Third International 1965, pp. 59 and 212 to 212, apart from a minor modifications for an inhomogeneous body of fluid, and the ASC report 08 122. 1. Numerical values are given in Appendix 1, the formulas in Appendix II and estimates for SRO in Appendix III.

2. Properties of Water

The Appendices reveal a number of simplifying characteristics:

a) Heat transfer must involve convection, conduction is far too slow.

b) Diffusion is negligible on the time scale of interest.

c) The thermosensitivity can be largely ignored.

d) Any negative temperature gradient is expected to induce turbulent convection, and therefore mixing.

e) Concentration gradients are likely to be as important as temperature gradients for driving convection.

f) The vertical vessel is better approximated as a thermal inverter on a time scale of a few days, a conductor on a time scale of months.

g) The electrical conductivity of salt solution suggests itself as a basis for monitoring the concentration and conductivity.

3. Steady State Operation

The predicted heat input to the 200 tons of H_2O plus 1000 tons of D_2O is 1MW from the rock walls and 2KW from the photovoltaic heater, which will be balanced by the heat removed by the separate cooling systems on the H_2O and D_2O recirculation system, respectively 100 and 100, per minute. In the recirculation flows were to enter the vessel at 1°C below the equilibrium temperature then the H_2O cooler would require 1MW and the D_2O cooler 2KW. The heat transfer within the bulk of the water is expected to be turbulent convection. In these circumstances most of the temperature drop between the heat sources and sinks can be expected to occur in the boundary layers with only a small, but > 10^-4°C/m from the local stability conditions temperature gradient within the main body of water. This gradient must unlikely be less than 10°C per m in the pure H_2O, the minimum necessary to satisfy the local conditions.
for convection (Appendix II), and 18–24°C per m should be more than adequate to drive turbulent convection in the D$_2$O.

4. Salination of D$_2$O

It is assumed that the NaCl will be stored as 50 m$^3$ of 5% brine and can be introduced at the flow rate of the recirculation system, 100 l per minute, in about 8 hours. If the brine, at higher density and lower temperature than the D$_2$O in the acrylic vessel, rises to the top of the vessel then turbulent convective mixing can be expected.

It is argued in Appendix III that the fluid dynamics within the D$_2$O will be effectively isolated by the acrylic vessel from the fluid dynamics of the H$_2$O. The convective time scale for a $\Delta T = 1°C$ is estimated (see Appendix III) as some minutes, but as the time varies only as $1/\sqrt{\Delta T}$ the actual value of $\Delta T$ is irrelevant.

It appears that the D$_2$O can be brought up to 2.25% NaCl concentration by introducing 20 m$^3$ of 5% NaCl concentration at 1°C over 8 hours, and extracting 50 m$^3$ with a mean concentration of 0.125%. Throughout the 8 hours the contents of the acrylic vessel is expected to have a uniform concentration varying linearly with time.

5. Deactivation of D$_2$O

It is proposed to remove the 0.25% salt in the D$_2$O by reverse osmosis, ideally rejecting 50 m$^3$ of the total of 1000 m$^3$ at a 5% concentration of salt. Seven days at 100 l/min are required to process 1000 m$^3$ and if no mixing occurs between the mixture of the 0.25% brine in the acrylic vessel and the purified D$_2$O returned to the vessel then deactivation will take just 7 days. If mixing does occur at a time constant $< 1$ day then deactivation will be exponential with a 7-day time constant.

The most favourable circumstances for mechanical stability, i.e., no mixing, is a positive temperature gradient and a negative concentration gradient, which could be achieved by pumping out the salt solution from the bottom of the acrylic vessel and returning the purified D$_2$O to the top of the vessel at a temperature a little higher, say 1°C, than the temperature of the contents of the acrylic vessel. This is equivalent to a power input of 4 kW for a flow rate of 90 l/min.

The estimates of Appendix III are very encouraging in the sense that it is most unlikely that convective mixing (and therefore temperature gradient) and a negative concentration gradient are established, independent of the (small) temperature gradients which will occur in the D$_2$O and are isolated from the D$_2$O by the acrylic vessel.

6. Conclusions

Recirculation of the D$_2$O, either or pure, for steady state operation might be carried out by alternating the temperature of the return flow to the top of the acrylic vessel between a few degrees of 4°C relative to the content of the vessel with a periodicity of the old day to cause convective mixing (time constant of minutes) and therefore homogeneity, while maintaining a steady mean temperature (time constant of 7 days). After the change, either to or from pure, it would be appropriate to set up the required temperature gradient by recirculating the D$_2$O at 4°C for some days.

The salination can be expected to take about 8 hours with the content of the acrylic vessel remaining homogeneous throughout, the return being maintained at -1°C for the 8 hours.
Derealization will require 3 days with the return at 6°C, plus a further 7 days at
-1°C to return the temperature to the desired value and ensure homogeneity through
convective mixing. During the first 7 days the salt concentration of salt will be spatially
inhomogeneous but possibly well defined. Premature homogeneity will be fully restored
in a fraction of a day by the 8°C return.

A caution must be entered with respect to turbulent convective mixing given the
calculous approach of the AGC report despite the huge magnitude of the Graham number.
Lamda and Elshitz are low compared, but exist nonetheless. If the heat transfer is indeed
via laminar convection it is likely that a few patterns of hexagonal cells will be formed,
flowing the centre and down the sides of each hexagon. The time constant for mixing
may then be determined by diffusion over a length with the magnitude of the dimension
of a hexagon, which depending to Lamda and Elshitz is a difficult theoretical problem.
The mixing time constant could be as long as the scale magnitude for diffusion of NaCl is
an inch in an hour, the distance varying by the square root of the time.

It will be necessary to monitor the concentration and homogeneity of the salt at
least to the extent of sensors in the input and output pipework. Electrical conductivity
shows the obvious technique and is simple enough that a sensing head might be mounted
with a calibration curve.

One real reservation should be mentioned concerning the local stability condition
set up in Appendix II for inhomogeneous brine. The thermodynamics is incomplete to
the extent that the chemical potential was ignored, but it is thought unlikely that this
will change the qualitative features.
Appendix I

Numerical Values of Parameters

The numbers below all refer to H$_2$O, not D$_2$O, as it is difficult to find a complete set for the latter and it is unlikely that there could be an important difference between the fluid dynamics of H$_2$O and D$_2$O.

1. Density of NaCl solution is linear in concentration c (g of NaCl per 100g of H$_2$O), $\rho = 1.16 \times 10^{-2}$ g/cm$^3$ at 20°C.

2. Diffusion of NaCl in H$_2$O, $D = 1.0 \times 10^{-4}$ m$^2$/s at 10°C and 0.03 g/m$^3$ per litre and is insensitive to concentration. $D = 0.025$ m$^2$/s. Characteristic time for a distance of 0.01 m is 10 years. Characteristic length for a time of 1 week is 0.5 m.

3. Volume thermal expansion $\beta = 210 \times 10^{-6}$/°C at 10°C.

4. Thermal conductivity of H$_2$O, $\kappa = 0.56$ W/m°K. A power transfer of 10KW distributed over a circular area of 22m diameter (206.3W per m$^2$) requires a gradient of 159 K per m.

5. Electrical conductivity $\sigma$ of NaCl solution of concentration c % (see 1.) is nearly linear in c at low c, $\sigma = 1.1 \times 10^{-1}$ ohm$^{-1}$ m$^{-1}$ per %.

6. Temperature dependence $\frac{d\sigma}{dc} = 0.005\kappa$ K$^{-1}$ at c = 20%.

7. Viscosity $\nu = 1.307$ m$^2$/s at 10°C. $\frac{d\nu}{dc} = 0.002$ K$^{-1}$ at 10°C.

8. Compressibility. Bulk modulus $K = 2.056$ GPa. For a pressure increase of 1.2 x 10$^6$Pa the fractional increase in density is 0.5 x 10$^{-4}$, an increase which would also be achieved by a temperature reduction of 0.288°K or a NaCl concentration of 0.008%.

9. Specific heat, $c_p = 4.18$ J/g°K

Water recirculated at 150 l/min and cooled 1°C below the reservoir temperature compensates to a power transfer of 0.71KW. The power associated with the kinetic energy of this water discharged through a pipe of 0.01m$^2$ cross-sectional area is 2.3 x 10$^{3}$W. If the recirculation cooler extracts 6.7KW, i.e., reduces the temperature 1°C, it will take a week to cool 10000 time by 1°C, assuming the acrylic vessel is thermally isolated.

10. Thermal Conductivity of Acrylic $\kappa = 0.17$ to 0.25 W/m°K.

The acrylic vessel with a thickness of 0.01 m and surface area of 0.02m$^2$ will transfer 4W/m$^2$ and a total of 1.8KW for a 1°C temperature difference. The time constant for reducing the temperature difference by acrylic conduction is therefore 4 weeks.
Appendix II

Convection

Laudau and Lifshitz give a number of relations concerning the mechanical stability of a fluid under gravity and the onset of laminar and turbulent convection. In particular they point out that if the temperature distribution is a function of the coordinates other than the vertical coordinate then mechanical equilibrium in the fluid is not possible, i.e. the fluid will exhibit macroscopic motion. (Ref 2 and 213).

For a fluid of uniform composition convection will occur for a temperature gradient,

\[
\frac{dT}{dz} < -\gamma g/\beta (\rho = 1.4 \times 10^{-4} \text{ K}^{-1} \text{ cm}^{-1} \text{ for water})
\]

where \(g\) is the acceleration of gravity. In the case of a concentration \(c\) (of salt) which varies with \(z\) it is necessary to modify the equations of Laudau and Lifshitz (Section 4, pp 8.6). A fluid element of specific volume \(V(p, z, c)\) at height \(z\) displaced (without mixing or heat conduction) to a height \(z + \Delta z\), where the pressure is \(p',\) entropy \(S\), and concentration \(c'\), will have a specific volume \(V(p', z', c)\), neglecting the small influence of \(p/p\) on \(c\), and displace an element of fluid of specific volume \(V(p', z', c)\). The condition for convection is

\[
V(p', z', c) - V(p, z, c) < 0
\]

i.e., \(\frac{dV}{dz} < 0\) which gives, ignoring any influence of the chemical potential,

\[
\frac{d\gamma}{dz} = \frac{T \gamma \beta}{\rho p c} + 1.4 \frac{dT}{dz}
\]

\[
= -1.4 \times 10^{-4} \frac{dT}{dz} + 34.1 \frac{dT}{dz}
\]

for water, where \(c\) is in % (g. of NaCl per 100g of H\(_2\)O. The concentration gradient clearly dominates the stability:

a) For a mean \(\delta < 0.25\%\) and \(c\) varying linearly over the 1m height, convection will occur for

\[
\frac{dT}{dz} < 6.42 \text{ C/m.}
\]

b) For \(c\) varying linearly from 0 to 0.25% over the one meter diffusion length of 24m convection will occur in the diffusion region for

\[
\frac{dT}{dz} < 2.55 \text{ C/m.}
\]

Laudau and Lifshitz also provide conditions for the onset of convection in a fluid bounded by planes at \(z = 0\) and \(z = l\) with a temperature difference \(\Delta T\), in terms of the Prandt, Grashof, and Rayleigh numbers \(P, G\), and \(Ra\) respectively.

\[
\frac{V}{\nu} = \frac{P}{\alpha T}\frac{\Delta T}{l^3}
\]


\[
G = \frac{\rho g l^3}{\nu^2} \Delta T^2 \text{ and } Ra = \frac{G}{P},
\]

where \(\alpha\) is the thermometric conductivity \(V\) is \(\nu/p\) is the kinematic viscosity.

Steady convection must occur for \(CP > 1710\), or 1100 for a free upper surface at \(T_1\), and turbulent convection sets in at \(G > 3 \times 10^9\). For water and \(l = 10\text{m}

\[
C = 1.2 \times 10^{12} \text{ C/m}
\]

and a temperature gradient \(\Delta T = 6 \times 10^{-3} \text{C/}\text{m}\) is sufficient to induce turbulence. The conditions for the onset of convection are for an incompressible fluid, but that is a good approximation for water.
Appendix III

Convective in H₂O

Zemansky and Van Dorn (ASC Report 90-1822-1, Appendix D, Dec. 1960), estimate the scale magnitudes for the H₂O as follows:

- Heat transfer: 10 ± 2 kW (given)
- Heat transfer: 2.5 kJ g⁻¹ (given)
- Length scale: 27 m (given)
- Temperature difference: ΔT = 1°C
- Grashof number: G = 10¹⁳
- Rayleigh number: Ra = 10¹⁴
- Characteristic time: Δt = 7 min
- Characteristic velocity: \( v = \frac{4}{3} \text{mm/s} \)

The convective time scale is given in the ASC report as:

\[
\Delta t = \frac{L^3}{\sqrt{G}} \left(1 + F^{-1}\right)
\]

\[
\approx \frac{L^3}{\sqrt{G}} \Delta T
\]

and the characteristic velocity as \( v = \Delta t / \Delta t \).

Turbulent convection in the H₂O is very probable (see however Section 6 of ASC report) and the ASC calculations indicate a temperature gradient in the H₂O adjacent to the acrylic vessel of a few millidegrees per minute. Both positive and negative gradients are obtained in the various calculations although it is pointed out that a negative gradient, which would tend to drive convection in the H₂O, is inherently unstable. If the gradient is negative, convection could occur transporting a few watts (limited by the thermal impedance of the acrylic) with a convective time scale equal of an hour, assuming of course that the recirculated H₂O does not set up a temperature gradient. The Grashof number is large, \( 10^{14} \), or presumably the convection would be turbulent.

If the recirculated H₂O were returned to the top of the acrylic vessel at 1°C relative to the bulk of the H₂O, turbulent convection would be expected with a time scale of minutes, similar to the H₂O. A positive concentration gradient of salt, resulting from recirculating the 1°C H₂O with a relatively high concentration of salt, can only enhance the convective mixing.

The converse situation in which (pure) H₂O is returned to the top of the acrylic vessel at 41°C relative to the bulk, has a positive temperature gradient and convection does not occur. This is a familiar situation in the laboratory and for that matter in any domestic hot water storage tank. If there is a negative concentration gradient that will also act against convection and should suppress convection even in the case of a negative temperature gradient of moderate magnitude.

It seems most unlikely that the H₂O temperature gradient of a few millidegrees per minute via the considerable thermal impedance of the acrylic vessel can invalidate the conclusion drawn for H₂O temperature gradients of \( \approx 100 \) millidegrees/min, or equivalent concentration gradients in time due to evaporation.