Data Analysis of ESC Counting for Ra and Rn

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1. Radon measurement in ESC

The electronic chamber measures radon in the following way; radon source is painted
to and is then introduced into the ESC with media gas. In some case, as for measuring thorium
and radium in the MnO2 beads, the media gas is circulated in a closed loop of chamber and
source.

Radon from source is decayed and about 3/4 of the daughter product, Po, are positively
charged. These Po ions are collected onto a Si alpha detector by an electric field applied between
the chamber and the detector. The detector detects the decays of Po and its further decay
products.

Th chain:

\[ ^{222}\text{Rn} \rightarrow ^{220}\text{Rn} \rightarrow ^{216}\text{Po} \rightarrow ^{212}\text{Pb} \rightarrow ^{208}\text{Pb} \]

U chain:

\[ ^{226}\text{Ra} \rightarrow ^{222}\text{Rn} \rightarrow ^{218}\text{Po} \rightarrow ^{214}\text{Po} \rightarrow ^{210}\text{Po} \rightarrow ^{206}\text{Pb} \]

The following alpha decays should be observed in an acquired spectrum if both sources are
presented:

<table>
<thead>
<tr>
<th>Radio nuclei</th>
<th>Energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Th chain:</td>
<td></td>
</tr>
<tr>
<td>$^{216}\text{Po}$</td>
<td>5772</td>
</tr>
<tr>
<td>$^{212}\text{Pb}$</td>
<td>6070</td>
</tr>
<tr>
<td>$^{212}\text{Po}$</td>
<td>8766</td>
</tr>
<tr>
<td>Uranium chain:</td>
<td></td>
</tr>
<tr>
<td>$^{218}\text{Po}$</td>
<td>6003</td>
</tr>
<tr>
<td>$^{214}\text{Po}$</td>
<td>7087</td>
</tr>
</tbody>
</table>

There should be three peaks from each chain. For the Po chain, the $^{216}\text{Po}$ is the direct
decay product of $^{220}\text{Rn}$. The calculation is straight forward and it is the main decay for $^{224}\text{Ra}$. 
$^{220}$Rn detection. Theoretically, the sum of $^{212}$Bi and $^{212}$Po will represent the $^{224}$Ra-$^{220}$Rn activity. The relative long life of $^{212}$Po causes non-equilibrium between $^{216}$Po and $^{212}$Bi+Po for entire counting. This complicates calculation. $^{212}$Bi peak is partly unclued into $^{218}$Po peak from uranium chain. The technique for separating two peaks for each other exists, but it's a time consuming task; $^{212}$Po decays extremely fast with a half life of 0.34s that is shorter than the electronics response time, some α is added into $^{217}$Bi. This leaves a long tail on higher energy side on the peak.

For U chain, $^{218}$Po peak is mainly used for $^{226}$Ra-$^{222}$Rn measurement. It needs only about 20 to 30 minutes to form a secular equilibrium with $^{222}$Rn. If high activity of $^{220}$Rn presents, the uses of the $^{214}$Po decay becomes essential. A few hours waiting are needed for the secular equilibrium between $^{222}$Rn and $^{214}$Po after the application of the electric field. $^{210}$Po peak is not generally seen or used as the 22 year half life of $^{210}$Po makes this peak useless.

2 Formalism

For a decay chain

$$A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \ldots$$

At $t=0$, if only $N_1$ exist and all daughter products are zero, at time $t$, $N_k(t)$ can be written as:

$$N_k(t) - N_k(0) = \sum_{j=1}^{k-1} \left( \frac{c^{-kt}}{c^{-t_1}} \right) \prod_{i=1}^{k-1} \left( \frac{t_i}{t_{i+1}} \right)$$

2.1. $^{224}$Ra analysis

From $t=0$ to $t$, there will be following amount of $^{224}$Ra decayed:

$$D(t) = N_{Ra}(0) - N_{Ra}(t) = N_{Ra}(0) \times (1 - e^{-t})$$

It needs only a few minutes to obtain a secular equilibrium up to $^{216}$Po. The latter should have exactly the same decay rate as $^{224}$Ra. The number of counts in $^{216}$Po alpha peak from $t=0$ to $t$ is the product of $D(t)$ and $C$:

$$C(t) = C \times D(t) = C \times N_{Ra}(0) \times (1 - e^{-t})$$

This is the so called cumulated data. If $C(t)$ and $t$ are recorded, and $e$ is well known, the $N_{Ra}$ can then be calculated with $N_{Ra}(0) = C(t)/(e \times (1 - e^{-t}))$. If time can be recorded accurately, the error of $N_{Ra}(0)$ can be written as:

$$dN_{Ra}/N_{Ra} = dC/C + dt/te$$
If the counting data between \(t_1\) to \(t_2\) is analyzed, \(C(t_1 \rightarrow t_2)\) would be \(C(t_2) - C(t_1)\):

\[
C(t_1 \rightarrow t_2) = e \cdot N_{Ra}(t_1) \cdot (\exp^{-\lambda_1 t_1} - \exp^{-\lambda_2 t_1})
\]

This is so called differential data analysis. \(N_{Ra}(0)\) and its error can be calculated in the same way as for cumulative data set. The differential data analysis is the correct analysis method for all Radon and Thoron measurement with ESC.

2.2 226Ra analysis

From \(t=0\) to \(t\), there will be:

\[
D_{Ra} = N_{Ra}(0) \cdot (1 - \exp^{-\lambda t})
\]

226Ra decayed and a same amount of 222Rn has been produced. The amount of 222Rn left at the end of time \(t\) should be:

\[
N_{Rn}(t) = N_{Ra}(0) \cdot \left( \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \exp^{-\lambda_2 t} \right)
\]

where \(\lambda_1\) is the decay constant of 226Ra and \(\lambda_2\) is that of 222Rn. The difference is the number of Rn decayed from \(t=0\) to \(t\):

\[
D_{Ra}(0 \rightarrow t) = D_{Ra}(0 \rightarrow t) - N_{Rn}(t) = N_{Ra}(0) \cdot (1 - \exp^{-\lambda_2 t} - \exp^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1} \exp^{-\lambda_2 t})
\]

At \(\lambda_1 = 5.5 \times 10^{-8}\) s\(^{-1}\) and \(\lambda_2 = 7.5 \times 10^{-5}\) s\(^{-1}\), and within our measurement time (<1000 hours), \(\exp(\lambda_1 t)\) will be always closed to 1.0, we can simplify \(\lambda_1 / (\lambda_2 - \lambda_1)\) to \(\lambda_1 / \lambda_2\) and \(\exp(-\lambda_1 t)\) to 1. \(\lambda_1\) t. It becomes:

\[
D_{Ra}(0 \rightarrow t) = N_{Ra}(0) \cdot (1 - (1 - \lambda_1 t) - \frac{\lambda_1}{\lambda_2} (1 - \exp^{-\lambda_2 t}))
\]

or

\[
D_{Ra}(0 \rightarrow t) = N_{Ra}(0) \cdot \lambda_1 t + \frac{\lambda_1^2}{2} - \frac{\lambda_1}{\lambda_2} (1 - \exp^{-\lambda_2 t})
\]

By ignoring the second order of \(\lambda_1\) that is extremely small in most circumstances, the final formula is:

\[
D_{Ra}(0 \rightarrow t) = A_{Ra}(0) \cdot (1 - \frac{1}{\lambda_2} (1 - \exp^{-\lambda_2 t}))
\]
with the activity of $^{226}$Ra. In practice, the Po decay is detected, and the efficiency of detection should be considered. Also if the $^{218}$Po counts between $t_1$ and $t_2$ is analyzed, the following formula is used:

$$C_{Po}(t_1 \rightarrow t_2) = \varepsilon \cdot A_{Po}(0) \cdot \left((t_2 - t_1) \cdot \gamma - \frac{1}{\lambda} \cdot (\exp(-\lambda t_1) - \exp(-\lambda t_2))\right)$$

The error of $A_{Ra}$ can be calculated in the same way as in the section 2.1. if time is recorded accurately.

2.3 Non-supported $^{222}$Ra analysis

Non-supported $^{222}$Ra can be Ra in air absorbed in beads or absorbed on surfaces of all components of the entire counting system. But its decay and counting is in the same way as $^{224}$Ra. The formulation in the section 2.1 can be applied here by replacing the $\lambda$ with that of $^{222}$Ra.

2.4 Fitting of data set

In presence of non-supported $^{222}$Ra, there would be two components in the same Po peak for $^{226}$Ra data analysis. The counts from $t_1$ to $t_2$ can be written as:

$$C(t_1 \rightarrow t_2) = \varepsilon \cdot A_{Po}(0) \cdot \left((t_2 - t_1) \cdot \gamma - \frac{1}{\lambda} \cdot (\exp(-\lambda t_1) - \exp(-\lambda t_2))\right) + \varepsilon \cdot N_{Po}(0) \cdot (\exp(-\lambda t_1) - \exp(-\lambda t_2))$$

At least a set of two $C(t_1 \rightarrow t_2)$ of different time is needed for the two unknown $A_{Po}(0)$ and $N_{Po}(0)$. The longer the counting time is performed and the more data points are recorded, the better the result would be.

3. Two practical analysis

3.1 $^{214}$Po peak replacing $^{218}$Po peak

As the $^{212}$Bi alpha has roughly the same energy as that of $^{218}$Po, in presence of high $^{224}$Ra activity, the analysis of $^{222}$Ra from $^{218}$Po decay becomes difficult. In this case, $^{214}$Po decay is used for the analysis of U chain. But $^{214}$Po is delayed by 26.8+19.8 minutes. We can prove that if $t_1$ is larger enough, say 6 hours, all three nuclei $^{214}$Po, $^{218}$Po and $^{222}$Ra will decay at the same rate. ($C(t_2)/C(t_1)$)z would be number of decays of all three. We just consider the first $t_1$ at $\geq6$ hours from a data set of hundreds hour counting.

3.2 In presence of Th

MnO$_2$ beads can extract Th from water as well as Ra. As $^{226}$Ra has a half life of 1602 years, the presence of $^{230}$Th will not interfere the analysis at all. But the presence of $^{228}$Th will. If we consider $^{228}$Th as $^{226}$Ra, and $^{224}$Ra as $^{222}$Ra in U chain, this is exactly the same situation.
in the section 2.4, we can say there is a supported $^{224}$Ra and a non-supported $^{224}$Ra. The same set of data can be fitted to obtain two unknown $A_{Rb}$ and $N_{Ra}$ using:

$$U(t_1 \rightarrow t_2) = e^{-A_{Rb}(t_2-t_1)} - e^{-A_{Rb}(t_2-t_1)} - \frac{1}{2} (e^{-A_{Rb}(t_2-t_1)} - e^{-A_{Rb}(t_2-t_1)}) + e^{-N_{Ra}(t_2-t_1)} (e^{-A_{Rb}(t_2-t_1)} - e^{-A_{Rb}(t_2-t_1)})$$

with the $\lambda$ of $^{224}$Ra.

4. Conclusion

Data analysis of Ra measurement is relatively easy comparing to the work. For an accurate radioactivity in water result, some points should be emphasized:

1) The background of the counting system as well as the beads and the column must be kept as low as possible. The data analysis of the system is the same as described above. For U, $^{228}$Ra and $^{222}$Rn activities are measured although the latter is not interesting. It can be very largely depending on environment. For Th chain, $^{224}$Ra component should not be seen in the system if it is contaminated or from newly made beads.

2) The entire water radio assay procedure should be standardized. Amount of beads used in each column should be constant; beads drying time should be longer enough and constant. The radon detection efficiency of the ESC should be verified once a year by using extremely weak source.

3) System should be sealed as tight as possible. Significant pressure raise can cause efficiency drop during measurement. Thus, incorrect result will be obtain.

4) Counting time of each measurement should be at least 100 hours. A longer counting time is preferred.
Energy Response and PMT Backgrounds in SNO
SNO-STR-96-015

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May 29, 1996

1 Introduction and Definitions

A beginning is a time for taking care that the balances are correct... --- Princess Isolde, The Manual of Moonlight [1]

Given the above advice, it seems prudent to start with a few general notes and definitions:

- Energies are kinetic energies in MeV.
- \( N_{\text{hit}} \) is the number of hits in an event.
- Unless otherwise stated, the true radius of an event has been used. This avoids any filter dependent complications (see discussion in Section 2).
- The mean and width of the distributions have been calculated by fitting a Gaussian to the \( N_{\text{hit}} \) spectrum. Although this is a better measure than calculating the mean and RMS spread, it still has problems with the 10 hit threshold, low energy (2 and 3 MeV) events tend to be somewhat distorted by this. The effect of this is to decrease the width and increase the mean \( N_{\text{hit}} \).
- SNOMAN 2.09 was used for this analysis (comparison with 2.08 and across the development in 2.09 shows the results to be stable).
- Where a point is given as the location of an event, the electrons are started at that point, where a radius is given the electrons are started in a thin spherically symmetric shell at that radius.
- The term "default detector configuration" refers to the default Monte Carlo configuration.

2 Energy Calibration

2.1 Default detector configuration (with and without NCDs)

The upper plot of Figure 1 shows the average \( N_{\text{hit}} \) for monoenergetic electrons and gamma as a function of the initial energy. The events were started uniformly throughout the D_2O. The lower plot of Figure 1 uses the same data, but shows \( N_{\text{hit}} \) per MeV. The energy response from sources capture on B and C is also shown. Table 1 shows some sample values. There are several points to note:

1
- The $N_{\text{obs}}$ plot is roughly linear in the region 2-5 MeV. However, the fit is not particularly good ($\chi^2$ of ~ 70 per degree of freedom). Extrapolating the line to 20 MeV leads to a substantial error, as can be seen from the figure. A much better fit is given by a quadratic ($\chi^2$ of ~ 1 per degree of freedom), which accounts for the fall off in $N_{\text{obs}}$ at low energies (due to the increasing importance of the Compton effect) and at higher energies (as the probability of a multiple strike in a tube becomes more important). The latter effect could presumably be dismissed by looking at the number of photo-electrons rather than $N_{\text{obs}}$, but this has not yet been investigated. Due to the poor charge resolution of the PMTs, it is not clear that the summation of deposited charge would improve the energy resolution of the detector.

- The extracted numbers are remarkably robust with respect to assumptions. Analysing the data under three scenarios (namely all recorded events, all events reconstructing within the D2O and all events reconstructing within 10 cm of its initial point) showed no statistically significant difference for either calibration or width. This suggests that the results here should be relatively stable with respect to choice of fit.

- The lower than expected energy response and greater width of the Chlorine gamma cascade is a result of the multiplicity of the gamma rays.

- The location of the event can have an effect in rare circumstances. Figure 2 shows the fit to the energy response of nC0 events at the centre of the detector, followed by two classical events and all events reconstructing inside 50 cm. The fits are visually identical, as are the fits made with other assumptions (such as only considering events which are reconstructed within 10 cm if the initial start point). Figure 3 shows the same analysis for events started at 50 cm. Events for which some of the photon escape into the light water form a high energy shoulder on the plot, which then tends to distort the Gamma fit. In general, this is not a problem, most events discussed in this section will not intrude into the light water, but it serves to highlight a potential pitfall.

- Extrapolate not shown in the figures. However, expected errors (for a 10,000 event sample) are ~ 1% for the width and ~ 0.1% for the mean.

Figure 4 shows the same plots but with the current SNUGEN (F.07) model of the NCDs inserted into the D2O. As has been predicted in other studies, the response of the detector drops by about 15%, but the general features are otherwise unaffected.

Figure 5 shows the percentage width as a function of initial energy with and without the NCDs. As expected, the NCDs increase the width of the $N_{\text{obs}}$ spectrum. Readers should note that the calculation of the width at higher energies can be complicated by the production of neutrons which subsequently capture on a detector producing a small series of events at ~ 5 MeV. In Monte Carlo this can be avoided by the simple expedient of turning the photo-distribution off, but this is not an easy optics in the real detector! The width distributions have a functional form of roughly $1/\sqrt{E}$, where $E$ is the kinetic energy, however, a better functional form is $P(1/E^2 - 1/E^2)$, where the coefficients are shown in Table 2.

2.1.1 Radial dependence

Table 2 shows a comparison of the mean $N_{\text{obs}}$ and width as a function of radius. Without the NCDs present, the $N_{\text{obs}}$ value only changes by about 2%, the drop at 39 cm reflects the fact that some of the electrons are entering the acrylic and producing less Compton light.
The percentage width is stable to within statistical error\(^1\). This is not the case with the NCDs included in the simulation. Here we see a strong dependence on radius in both the magnitude and the width of the distribution. The change in the magnitude may be explained by remembering that events which occur at the centre of the detector see a forest of NCDs regardless of orientation. Events that occur in the outer parts of the D\(_2\)O, heading out, see only a few NCDs, whilst those heading into the detector are the above noted forest. This can be shown to give rise to a higher average, and also illustrates why the width increases as a function of radial distance.

2.2 Directional dependence and NCDs

The most used directional dependence that occurs with the introduction of the NCDs appears to be overestimated (see Table 4), at least in the cases studied. The largest deviations noted were of order 15\(^\circ\). This relative insignificance of this effect appears to be a result of multiple scattering and the large opening angle of the Compton cone. At these energies, the concept of an electron traveling parallel or perpendicular to the length of the NCDs is not valid. The only effect of interest is that the width for events at 150\(^\circ\) does have a directional dependence, and one that would become more exaggerated if events at the edge of the detector headed out were compared with those headed in.

2.3 The effect of changing the detector configuration

The Table 5 shows the effect on the N\(_{\text{low}}\) spectrum for various changes to the detector configuration. The width is not shown as it was found to scale with the mean N\(_{\text{low}}\). As can be seen from the table, all the effects noted cause a 1−2\% change in the mean N\(_{\text{low}}\), a change that seems to be essentially independent of the location of the event. The exception is the addition of the tils, which has little effect. Combining the tils and the belly plates give rise to a roughly 1.3\% drop in detector efficiency, regardless of energy. This means that the calibration figures given elsewhere in this report may have a systematic overestimate on them.

2.4 Conclusions

SNO\(_{\text{MAX}}\) is predicting \(\sim 16.5\) ktons per MeV under normal circumstances and \(\sim 8.7\) ktons with the NCDs in. These numbers have a systematic error of \(\sim 1.5\%\) due to the presence in the real detector of the belly plates and welded joints at the periphery of the acrylic tils. The number of ktons is essentially linear with energy, though at 20 MeV a slight curvature is noted, arising from the increasing probability of multiple photons scoring a hit on the same photomultiplier. The presence of the NCDs has a definite impact on the dependence of N\(_{\text{low}}\) with radius, but little on the directional dependence of the calibration. The width of the distribution does carry a directional dependence. At low energies, the relation between N\(_{\text{low}}\) and energy is non-linear, due to the effects of the Compton cutoff and the hardware threshold of \(\sim 15\) ktons.

3 PMT Backgrounds

To date only \(^{210}\)Po, \(^{222}\)R\(_{\text{a}}\) and \(^{228}\)Th have been examined - these decays have the highest Q-values by some margin. The current status is shown in Table 6. The events were set up inside

\(^1\)Error of 0.1\% in the mean and 1\% in the width are expected from a sample of 1,000,000 events.

\(^2\)And it should be noted that this is using the actual incident direction of the electron. The uncertainty in the calculated direction in real data may serve to wash out this to some extent.

3
the PMT glass bulb, and the threshold was set at 30 hits for reasons of disk space. It should be noted that this distorts the spectrum near the threshold. The time filter was used in the analysis (as a worst case estimator), and the number of hits per pulse-tube was used as an indication of the energy. The number of events per year was derived from assuming the tube's activity to be evenly distributed through the PMT glass bulb (the neck was ignored), taking a glass mass of 468 g, and an average of the Geissler and Kirkhope measured activities, taken from [5]. Equilibrium conditions were assumed.

The numbers in Table 6 suggest that there are no PMT events above 5 MeV (the highest recorded energy was a single event at 53 hits), whilst the highest energy reconstructing into the D0 was 68 hits, corresponding to ~ 1.3 MeV. There are however, ~ 1500 events per year reconstructing inside the D0 with energies in the range 3.4 MeV, and roughly 200 events per year, inside 4m. It should be noted that the statistics on these numbers is poor; most of the 500 events inside 4m come from the pair of 205Tl events that reconstructed, multiplied by the scaling factor of 100.

The numbers are also strongly filter dependent. The 'grid filter' line of Table 6 shows a re-analysis of the 205Tl data with the grid filter. The number reconstructing into the D0 falls by a factor of 3, though the tail of the distribution still goes out to a little over 40 hits. This is not to say that the grid filter is the ideal tool for the job, but it does serve to highlight the fact that exact predictions of PMT beta-gamma are in the same league as ten-leaf reading. A factor of two error (from choice of filter) in the predicted numbers of PMT beta-gamma would seem conservative.

The NCDs' line shows what happens when the NCDs are introduced into the detector. Essentially, the rate drops by a factor of 2, but the tail reconstructing into the D0 still goes out to 42 hits; now roughly 5 MeV. No attempt has been made to apply the radial dependence energy calibration to the PMT beta-gamma. The average Ntot per MeV is higher, it is the outer part of the D0, where most of the beta-gamma reconstructions, which would serve to reduce this problem somewhat.

5.1 Conclusions

A study of the PMT beta gamma singles and doubles rates suggests that of order 1500 events per year will reconstruct into the D0 over a threshold of 30 hits. This prediction for the number of events is subject to a number of caveats, the most important being that the quality of the filter may reduce this by at least a factor of two, and that most of the predicted events come from the extrapolation of the 205Tl run, which currently has exceptionally poor statistics. The distribution goes out to 45 hits, corresponding to approximately 4 MeV, and has no real impact on the study of solar neutrino events above a threshold of 5 MeV, it is not clear to the author whether this will have an impact on other, lower energy, studies.

The addition of the NCDs lowers the number of reconstructing events by a factor of approximately 3, but the tail still extends out to 42 hits, corresponding to approximately 5 MeV, and thus still of no real concern, at least for solar neutrino events.

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*This is thought to be superior to looking at the number of photo-electrons in it, avoids the meaning of the signal that the single photodecay is major, would add.

*The event rate is a strong function of Ntot. While the maximum observed hit is only a guide to the end point, it is the author's (variants) opinion that events more that a couple of Ntot higher than the numbers are very unlikely. There is no current evidence to support a long tail in high Ntot.
References

Table 1: Detector response for sample event classes started isotropically in the D$_2$O.

<table>
<thead>
<tr>
<th>Event</th>
<th>Response (N$_{hit}$)</th>
<th>Width (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 MeV e$^-$.</td>
<td>51.82</td>
<td>17.3</td>
</tr>
<tr>
<td>10 MeV e$^-$.</td>
<td>105.3</td>
<td>12.9</td>
</tr>
<tr>
<td>a0</td>
<td>53.82</td>
<td>17.3</td>
</tr>
<tr>
<td>aC</td>
<td>61.17</td>
<td>22.6</td>
</tr>
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Table 2: Parameters for fitted functional forms of the width.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
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<tbody>
<tr>
<td>Electrons</td>
<td>42.35 ± 2.70</td>
<td>0.76 ± 0.57</td>
<td>0.26 ± 0.18</td>
</tr>
<tr>
<td>Gammas</td>
<td>54.62 ± 1.09</td>
<td>2.17 ± 0.42</td>
<td>0.25 ± 0.18</td>
</tr>
<tr>
<td>Electrons (NCDs)</td>
<td>44.36 ± 2.20</td>
<td>3.87 ± 0.86</td>
<td>2.09 ± 0.35</td>
</tr>
<tr>
<td>Gammas (NCDs)</td>
<td>47.87 ± 2.33</td>
<td>9.65 ± 0.82</td>
<td>4.46 ± 0.82</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>No NCDs</th>
<th>Width</th>
<th>With NCDs</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50.62 (1.05)</td>
<td>5.50 (16.6)</td>
<td>41.33 (1.00)</td>
<td>7.35 (17.8)</td>
</tr>
<tr>
<td>100</td>
<td>59.92 (1.00)</td>
<td>8.67 (7.0)</td>
<td>48.15 (0.97)</td>
<td>5.72 (18.7)</td>
</tr>
<tr>
<td>200</td>
<td>51.61 (1.05)</td>
<td>8.90 (6.6)</td>
<td>41.81 (1.00)</td>
<td>7.65 (12.1)</td>
</tr>
<tr>
<td>300</td>
<td>52.99 (1.34)</td>
<td>8.91 (6.3)</td>
<td>47.49 (1.00)</td>
<td>8.29 (10.5)</td>
</tr>
<tr>
<td>400</td>
<td>53.07 (1.06)</td>
<td>9.07 (6.9)</td>
<td>44.28 (1.07)</td>
<td>8.91 (20.1)</td>
</tr>
<tr>
<td>500</td>
<td>53.71 (1.04)</td>
<td>9.12 (7.6)</td>
<td>46.30 (1.12)</td>
<td>9.63 (20.5)</td>
</tr>
<tr>
<td>600</td>
<td>52.36 (1.00)</td>
<td>9.08 (7.0)</td>
<td>47.25 (1.14)</td>
<td>8.65 (20.4)</td>
</tr>
<tr>
<td>700</td>
<td>52.98 (1.03)</td>
<td>9.08 (7.1)</td>
<td>47.40 (1.14)</td>
<td>9.76 (20.6)</td>
</tr>
<tr>
<td>800</td>
<td>50.51 (0.89)</td>
<td>8.36 (16.6)</td>
<td>40.15 (1.12)</td>
<td>9.14 (19.8)</td>
</tr>
</tbody>
</table>

Table 3: Mean and width of N\textsubscript{NN}, as a function of radial distribution. For the mean, the figures in brackets show the value relative to the first row. For the width columns, the number in brackets is the percentage width.

<table>
<thead>
<tr>
<th>Directional</th>
<th>5 MeV</th>
<th>10 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Container</td>
<td>N\textsubscript{NN}</td>
<td>Width</td>
</tr>
<tr>
<td>isotropic</td>
<td>Events at 50 cm</td>
<td>41.15 (1.00)</td>
</tr>
<tr>
<td>(0, 0)</td>
<td></td>
<td>41.24 (1.001)</td>
</tr>
<tr>
<td>(0, 0)</td>
<td></td>
<td>41.46 (1.007)</td>
</tr>
<tr>
<td>(0, 0)</td>
<td></td>
<td>41.07 (0.996)</td>
</tr>
</tbody>
</table>

| Events isotropic in\ D\textsubscript{2}O |
| isotropic   | 45.15 (1.000) | 9.43 | 90.66 (1.000) | 15.16 |
| (0, 0)      | 45.41 (1.005) | 9.54 | 90.92 (1.000) | 15.06 |
| (0, 0)      | 45.11 (0.999) | 9.55 | 90.51 (0.999) | 13.42 |
| (0, 0)      | 44.53 (0.998) | 9.29 | 90.68 (0.998) | 13.55 |

| Events at 559 cm |
| isotropic   | 47.25 (1.000) | 9.35 | 68.82 (1.000) | 15.85 |
| (0, 0)      | 47.41 (1.003) | 9.73 | 94.31 (1.000) | 15.69 |
| (0, 0)      | 46.67 (0.998) | 9.24 | 93.05 (0.992) | 15.64 |
| (0, 0)      | 46.83 (0.999) | 9.85 | 93.39 (0.998) | 16.13 |

Table 4: Directional dependence for events started in the\ D\textsubscript{2}O with energies of 5 and 10 MeV. The special distribution consists of events at 65° to the z-axis. The numbers in brackets show the ratio of N\textsubscript{NN}, to the angular isotropic case. NCDs are included in the simulation. Statistical errors on N\textsubscript{NN}, are ~0.1%.
Table 5: Calibration changes under a number of circumstances. The electrons were started with an isotropic angular distribution. Runs (1) and (2) were started isotropically in the D_2O. Run (3) was started at the origin, Run (4) was started at 550 cm. The attenuation lines had the appropriate attenuation coefficient increased by 20%. Statistical errors on N_{att} are ~0.1%, and on the width are ~1.5%.

<table>
<thead>
<tr>
<th>Source</th>
<th>Rate</th>
<th>Run in D_2O</th>
<th>in air</th>
<th>Total Scale</th>
<th>In D_2O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Fitter - PMT angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>206Pb11</td>
<td>7.5(5)</td>
<td>7.5(7)</td>
<td>7.5(1)</td>
<td>7.5(7)</td>
<td>3.0(7)</td>
</tr>
<tr>
<td>206Rb1</td>
<td>3.0(5)</td>
<td>3.0(5)</td>
<td>3.0(5)</td>
<td>3.0(5)</td>
<td>3.0(5)</td>
</tr>
<tr>
<td>205Rb1</td>
<td>1.9(5)</td>
<td>1.9(5)</td>
<td>1.9(5)</td>
<td>1.9(5)</td>
<td>1.9(5)</td>
</tr>
<tr>
<td>Time Fitter - SDC doubles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>206Pb11</td>
<td>96(5)</td>
<td>96(5)</td>
<td>96(5)</td>
<td>96(5)</td>
<td>0</td>
</tr>
<tr>
<td>206Rb1</td>
<td>2.4(4)</td>
<td>2.4(4)</td>
<td>2.4(4)</td>
<td>2.4(4)</td>
<td>13(4)</td>
</tr>
<tr>
<td>205Rb1</td>
<td>1.6(5)</td>
<td>1.6(5)</td>
<td>1.6(5)</td>
<td>1.6(5)</td>
<td>1.6(5)</td>
</tr>
<tr>
<td>205Rb1</td>
<td>1.0(4)</td>
<td>1.0(4)</td>
<td>1.0(4)</td>
<td>1.0(4)</td>
<td>1.0(4)</td>
</tr>
<tr>
<td>Grid Filter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>206Pb11</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
</tr>
<tr>
<td>205Rb1</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
<td>3.9(7)</td>
</tr>
</tbody>
</table>

Table 6: PMT δ = γ rate for single events and double coincidence events. Columns one and two show the source decay and the expected rate. Columns 3 to 6 show the number of events run, the number reconstructed inside the D_2O and inside 4m, and the total number of events over threshold. Columns 7 and 8 show the scaling factor to 1 year, and the expected number of events reconstructing in the D_2O per year. The numbers in brackets are the maximum N_{att} in any category, and are presented to give the flavour of how far the spectrum extends.
Table 7: Prototype activity measurements, given in ppb by mass.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>39Ar</th>
<th>39Ar/Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>$47.6\pm 4.3$</td>
<td>$18.3\pm 3.5$</td>
</tr>
<tr>
<td>Bremsstrahlung</td>
<td>$38.8\pm 4.6$</td>
<td>$16.1\pm 1.5$</td>
</tr>
<tr>
<td>Average</td>
<td>$43.7$</td>
<td>$15.0$</td>
</tr>
</tbody>
</table>

Figure 1: SNO calibration as a function of energy. Electrons isotropically in the D$_2$O, no KCDs.
Figure 2: Gaussian fits to the $N_{	ext{tot}}$ response for $\pi$Cl $\gamma$ cascades isotropically generated in the D$_2$O. See text.

Figure 3: Gaussian fits to the $N_{\text{tot}}$ response for $\pi$Cl $\gamma$ cascades generated at a radius of 290 cm in the D$_2$O. See text.
Figure 4: SNO calibration as a function of energy for electrons created isotropically in the D$_2$O. The NCs are included in the simulation.
Figure 5: $N_{	ext{max}}$ width as a function of energy for electrons and gammas created isotropically in the D$_2$O. The lines are merely to guide the eye and have no other significance. The drop-off at low energies represents an artifact from the 10 bit cutoff.
SNO-STR-96-055
Fitting Optical Constants in SNO
Using a Pulsed N₂/Dye Laser

Richard Ford (Queen's)
March 2, 1997

Abstract

This report outlines methods under investigation to enable an on-going monitoring of the SNO detector optical condition using the pulsed nitrogen dye laser and diffuser ball system. The absorption lengths in the H₂O, acrylic and H₂O are to be determined by extracting NHT with various time cuts from the cumulative pmt time histograms. Bulk scattering, pmt QE and reflector condition are also to be determined from analysis of time histograms.

1 Introduction

Understanding the optical condition of the SNO detector is critical to the analysis the detector physics. The detector response to electrons, gamma rays and neutrons can only be measured at certain energies and locations and under limited conditions. Since light transmittance to the pmt is a function of event position we expect to rely on Monte Carlo modeling for the analysis. Also the detector gain and response could vary unpredictably due to changes in response of the pmt or changes in the detector optical condition. These changes could include varying impurities in the water systems, deposits on the acrylic or pmt reflectors or gain and efficiency drifts in the pmt. All the optical parameters are wavelength dependent so that the Cherenkov spectrum will evolve as the light travels through the detector.
The optical parameters must be determined at several wavelengths so that this extra positional dependence is correctly modeled in the Monte Carlo (SNOMAN).

This report describes on-going work to develop techniques and analyze methods to determine and monitor critical optical parameters. The recent calibration review [4] emphasized the need for such an interim document outlining progress in the development of optical calibration procedures. This report describes how the optical parameters will be monitored by global fits to time histogram data. Another document [1] reports how some of the techniques described here will be used to perform the initial optical characterization of the SNO detector by boot strapping from air-fill data. This is important, as a measurement of the light attenuation and scattering in the acrylic tiles can only be done once during air-fill before they become complicated by covariances with light water and bulk scattering effects.

The report begins by describing the optics of the SNO detector and the uncertainty with which the optical parameters must be determined. We then describe the optical source, how the detector is triggered and the data acquisition using pmt timing histograms. Next we move through the data processing and, finally, we present procedures that are being developed for fitting to extract the optical parameters. As this work is still on-going a complete calibration run-through including global fit to all optical parameters is not yet completed. Issues dealing with the assessment of bulk scattering and the pmt concentrator reflectors are also discussed.

2 Optical Parameters

2.1 Detector Optical Parameters

To completely understand the optical response of the detector we need to be able to determine, at any time, the following optical parameters.

- Attenuation lengths of the D$_2$O, the acrylic and the H$_2$O at several wavelengths between 300nm and 450nm. We would want to know these attenuation lengths for each acrylic panel, any across panels variations and the effects of rope grooves and sanded joints. The attenuation

\[ \text{1As of December 1996, it seems that there likely will not be any sanded joints. All bond areas are now fine polished to aid inspections.} \]
lengths in the water should be known at as many locations as necessary to account for possible non-homogeneous regions.

- Bulk scattering lengths (cross-sections) and the angular distributions of scattering for all detector regions and at several wavelengths.

- The position/angular response of the pmts and relative quantum efficiencies (QE) of all the pmts.

- Detector optical gain response (absolute QE).

In reality it will be impossible determine all of these parameters and distributions. Instead we must determine which parameters are important and how to measure them, and then understand any limitations this imposes on the analysis. The most critical optical parameters are the extinction coefficients (absorption plus scattering) in the detector media and the relative quantum efficiencies. By deploying the light source $P_0$, at certain locations the separate acrylic panels and water regions can be inspected. Cuts on the pmt timing information can be used to separate out scattered and multiple-reflected light. Other sources (e.g. $^{16}$N) can provide the overall gain figure.

2.2 Parameter Uncertainty Estimates

For the laserball at the centre of the detector the $D_2O$ attenuates the the total integrated light by a factor $e^{-rR}$ where $R$ is the inner acrylic radius. For off-centre locations at radius $r$ the attenuation is

$$I_r = \frac{1}{2} \int_{-1}^{1} \exp \left( -\mu r \cos \theta - \mu_1 \sqrt{R^2 + (1 - \cos^2 \theta)} \right) d(\cos \theta)$$

(1)

Evaluating this integral for various $r$ one finds that with a $D_2O$ attenuation length of 40cm there is up to a 5% increase in light when the source is near the acrylic. If systematic shifts in the energy calibration are to be less than 1% then we want the $D_2O$ extinction coefficient to be determined to an uncertainty of about 20%.

Now the attenuation lengths are wavelength dependent, especially near the UV, so we really want to weight the integral in equation (1) for the Cherenkov spectrum ($\lambda^{-\gamma}$) and the pmt quantum efficiency (QE($\lambda$)). In [6] this calculation has being done by averaging over the wavelength spectrum.
(see fig 1). One finds that the integral varies most rapidly for the acrylic with the inference that we need to determine the acrylic attenuation to better than 5%. For the D₂O the required uncertainty is better than 20% while for H₂O 25% uncertainty should be sufficient [6].

It is important to note the approximations assumed in [8]. Sheet-to-sheet variations in acrylic attenuation are not included. Reflections and refraction are not included, both of which would tend to increase the light variation, with the refraction effect being dependent on the acrylic attenuation. Also, we probably want to calculate these uncertainty estimates for each wavelength rather than average over the spectrum. Lastly, isotropic light is assumed while Cherenkov light is directional, thus these estimates are only true in the high statistics limits that permit a reasonable directional average for each radius. These calculations need to be performed with more detailed geometry and scattering effects using Monte Carlo (SNOMAN).

We note also that the variation in the integral (1) has the opposite sign between D₂O and acrylic, and the same sign between acrylic and H₂O. Co-variances between parameter uncertainties for the D₂O and acrylic will require care in assessing the detector positional energy dependence. This will not be as bad for co-variances between acrylic and H₂O parameter uncertainties, which is fortunate as it is known that these co-variances will exist due to some degeneracy in the light path distances. Effects due to scattering have the same sign as for acrylic attenuation (opposite to D₂O attenuation).

Measured data for the acrylic light transmission is presented in figure 2. The panel to panel variations are about 10%. In figure 3 I show the distribution of total integrated detectable light for these data for a Cherenkov source at the centre of the vessel. The Cherenkov output is multiplied by the acrylic transmission and the pmt quantum efficiency and integrated from 250nm to 600nm. The variation is about 1-2%, except that there is a tail extending to 5%. This tail is identified with the panels that have distinctly higher cutoffs² as seen with the distributions in figure 2.

²Although I have not yet tracked down the identity of all the panels, it is believed that the panels in the tail of the distribution with the higher cutoffs are from the first production run and were used in the AV qualification wall.
Figure 1: The upper plot is the total attenuation as a function of source radius for an isotropic light source. The second plot is the wavelength integrated attenuation. From [6] using white book data [7].
Figure 2: The top two plots show the total transmission and attenuation coefficient as a function of wavelength for a 5.84 cm acrylic panel sample. The remaining plots show the distribution of attenuations for all panels measured at various wavelengths.
Figure 3: (a) Cherenkov output, pent response and acrylic transmission as a function of wavelength. (b) The solid line shows the effective QE by including the AV transmission from the data in the first plot. The dashed line is weighted by the Cherenkov spectrum. (c) The bottom plot shows the distribution of detectable Cherenkov light by integrating over the data in plot (b) for each of the acrylic measurements. The light scale is arbitrary.