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Fitting Optical Constants in SNO
Using a Pulsed $\text{N}_2^*$/Dye Laser

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Abstract

This report outlines methods under investigation to enable an ongoing monitoring of the SNO detector optical condition using the pulsed nitrogen dye laser and diffuser ball system. The absorption lengths in the $\text{D}_2\text{O}$, acrylic and $\text{H}_2\text{O}$ are to be determined by extracting NMT with various time bins from the cumulative pmt time histograms. Bulk scattering, pmt QE and reflector condition are also to be determined from analysis of time histograms.

1 Introduction

Understanding the optical condition of the SNO detector is critical to the analysis the detector physics. The detector response to electrons, gamma rays and neutrons can only be measured at certain energies and locations and under limited conditions. Since light transmission to the pmts is function of event position we expect to rely on Monte Carlo modeling for the analysis. Also the detector gain and response could vary unpredictably due to changes in response of the pmts or changes in the detector optical condition. These changes could include varying impurities in the water systems, deposits on the acrylic or pmt reflectors or gain and efficiency drifts in the pmts. All the optical parameters are wavelength dependent so that the Cherenkov spectrum will evolve as the light travels through the detector.
The optical parameters must be determined at several wavelengths so that this extra positional dependence is correctly modeled in the Monte Carlo (SNOMAN).

This report describes on-going work to develop techniques and analysis methods to determine and monitor critical optical parameters. The recent calibration review [4] emphasized the need for such an interim document outlining progress in the development of optical calibration procedures. This report describes how the optical parameters will be monitored by global fits to time histogram data. Another document [1] reports how some of the techniques described here will be used to perform the initial optical characterization of the SNO detector by boot strapping from air-fill data. This is important, as a measurement of the light attenuation and scattering in the acrylic tiles can only be done once during air-fill before they become complicated by covariances with light water and bulk scattering effects.

The report begins by describing the optics of the SNO detector and the uncertainty with which the optical parameters must be determined. We then describe the optical source, how the detector is triggered and the data acquisition using pmtn timing histograms. Next we move through the data processing and, finally, we present procedures that are being developed for fitting to extract the optical parameters. As this work is still on-going a complete calibration run-through including global fits to all optical parameters is not yet completed. Issues dealing with the assessment of bulk scattering and the pmtn concentrator reflectors are also discussed.

2 Optical Parameters

2.1 Detector Optical Parameters

To completely understand the optical response of the detector we need to be able to determine, at any time, the following optical parameters.

- Attenuation lengths of the D$_2$O, the acrylic and the H$_2$O at several wavelengths between 300nm and 450nm. We would want to know these attenuation lengths for each acrylic panel, any across panels variations and the effects of rope grooves and sanded joints. The attenuation

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1As of December 1998, it seems that there likely will not be any sanded joints. All bond areas are now fine polished to aid inspections.
lengths in the water should be known at as many locations as necessary to account for possible non-homogeneous regions.

- Bulk scattering lengths (cross-sections) and the angular distributions of scattering for all detector regions and at several wavelengths.

- The position/angular response of the pmts and relative quantum efficiencies (QE) of all the pmts.

- Detector optical gain response (absolute QE).

In reality it will be impossible determine all of these parameters and distributions. Instead we must determine which parameters are important and how to measure them, and then understand any limitations this imposes on the analysis. The most critical optical parameters are the extinction coefficients (absorption plus scattering) in the detector media and the relative quantum efficiencies. By deploying the light source [2] at certain locations the separate acrylic panels and water regions can be inspected. Cuts on the pmt timing information can be used to separate out scattered and multiple-reflected light. Other sources (e.g. 161Y) can provide the overall gain figure.

2.2 Parameter Uncertainty Estimates

For the laserball at the centre of the detector the D$_2$O attenuates the total integrated light by a factor $e^{-\mu R}$ where $R$ is the inner acrylic radius.

For off-centre locations at radius $r$ the attenuation is

$$\frac{L}{L_0} = \frac{1}{2} \int_{-1}^{1} \exp \left( -\mu r \cos \theta - \mu \sqrt{r^2 + (1 - \cos^2 \theta)} \right) d(\cos \theta)$$

(1)

Evaluating this integral for various $r$ one finds that with a D$_2$O attenuation length of 40m there is up to a 5% increase in light when the source is near the acrylic. If systematic shifts in the energy calibration are to be less than 1% then we want the D$_2$O extinction coefficient to be determined to an uncertainty of about 20%.

Now the attenuation lengths are wavelength dependent, especially near the UV, so we really want to weight the integral in equation (1) for the Cherenkov spectrum ($\lambda^{-2}$) and the pmt quantum efficiency ($QE(\lambda)$). In [6] this calculation has been done by averaging over the wavelength spectrum.
(see fig 1). One finds that the integral varies most rapidly for the acrylic with the inference that we need to determine the acrylic attenuation to better than 5%. For the D₂O the required uncertainty is better than 20% while for H₂O 25% uncertainty should be sufficient [6].

It is important to note the approximations assumed in [6]. Sheet-to-sheet variations in acrylic attenuation are not included. Reflections and refraction are not included, both of which would tend to increase the light variation, with the refraction effect being dependent on the acrylic attenuation. Also, we probably want to calculate these uncertainty estimates for each wavelength rather than average over the spectrum. Lastly, isotropic light is assumed while Cherenkov light is directional, thus these estimates are only true in the high statistics limits that permit a reasonable directional average for each radius. These calculations need to be performed with more detailed geometry and scattering effects using Monte Carlo (SNOMAN).

We note also that the variation in the integral (1) has the opposite sign between D₂O and acrylic, and the same sign between acrylic and H₂O. Co-variances between parameter uncertainties for the D₂O and acrylic will require care in assessing the detector positional energy dependence. This will not be as bad for co-variances between acrylic and H₂O parameter uncertainties, which is fortunate as it is known that these co-variances will exist due to some degeneracy in the light path distances. Effects due to scattering have the same sign as for acrylic attenuation (opposite to D₂O attenuation).

Measured data for the acrylic light transmission is presented in figure 2. The panel to panel variations are about 10%. In figure 3 I show the distribution of total integrated detectable light for these data for a Cherenkov source at the center of the vessel. The Cherenkov output is multiplied by the acrylic transmission and the pmt quantum efficiency and integrated from 250nm to 600nm. The variation is about 1-3%, except that there is a tail extending to 5%. This tail is identified with the panels that have distinctly higher cutoffs\(^2\) as seen with the distributions in figure 2.

\(^2\)Although I have not yet tracked down the identity of all the panels, it is believed that the panels in the tail of the distribution with the higher cutoffs are from the first production runs and were used in the AV qualification wall.
Figure 1: The upper plot is the total attenuation as a function of source radius for an isotropic light source. The second plot is the wavelength integrated attenuation. From [6] using white book data [7].
Figure 2: The top two plots show the total transmission and attenuation coefficient as a function of wavelength for a 5.84 cm acrylic panel sample. The remaining plots show the distribution of attenuations for all panels measured at various wavelengths.
Figure 3. (a) Cherenkov output, pmt response and acrylic transmission as a function of wavelength. (b) The solid line shows the effective QE by including the AV transmission from the data in the first plot. The dashed line is weighted by the Cherenkov spectrum. (c) The bottom plot shows the distribution of detectable Cherenkov light by integrating over the data in plot (b) for each of the acrylic measurements. The light scale is arbitrary.
2.3 NHIT Dependence on Optical Parameters

For the laserball at position \( \vec{L}_i \), over any time the number of hits in PMT \( i \) (position \( \vec{P}_i \)) due only to the light pulses is given by

\[
\text{NHIT}_{ij} = RN_{ij} \Omega_{ij} e^{-\frac{S_{ij}}{\lambda_{ij} \rho_{ij}}} \times \exp \left( -x_{ij}^{\text{water}} \lambda_{ij} \rho_{ij} - \sum_{k=1}^{N_{\text{acrylic}}} x_{ij}^{\text{acrylic}} \lambda_{ij} \rho_{ij} \right) 
\times (1 - R_{ij})(1 - R_{ij})(1 + S_{ij})
\]

(2)

where

- \( R \) = Number of laserball pulses
- \( N_{ij} \) = Photons per pulse into unit solid angle
- \( \Omega_{ij} \) = Solid angle subtended
- \( e_{ij} \) = QE including pmt angular response
- \( \lambda \) = Extinction coefficient
- \( x_{ij}^{\text{water}} \) = Distance through D_2O or H_2O
- \( x_{ij}^{\text{acrylic}} \) = Distance through acrylic panel \( k \). Note the sum is just for convenience since for a particular \( i \) and \( j \) only one \( x_{ij}^{\text{acrylic}} \) is non zero.
- \( R_{\text{water}} \) = Water-acrylic reflection coefficients
- \( S_{ij} \) = Hits due to scattered light

In general the extinction coefficients might also depend on \( \vec{L}_i \) and \( \vec{P}_j \) due to inhomogeneities within the water volumes.

The factor \( S_{ij} \) that adds back the detected scattered intensity is a very loose approximation since this does not depend only on the positions \( L \) and \( P \), but it is a complicated function of the entire detector. This is the bane of all optical photometry experiments. The size of the SNO detector makes it unique as an optical instrument, since an analysis of the arrival times of the pm hits permits the factor \( S_{ij} \) to be partially separated out.

Figure (4) contains Monte Carlo histograms of the time between the laser pulse and the time when pmts are hit. The laserball light intensity is low so that all pm hits result from single photo-electrons (spe). Here the laserball is positioned at the centre of the D_2O, the first histogram is summed over all pmts and the second is typical for a single tube.
Figure 4: (a) Summed time histogram for laserball at centre of the vessel. (b) Single channel histogram (pmt 006x). Data are from SNOMAN for 10,000 events of 1855 ± 21 eV photons with the complete geometry and the 3D pmt model.
The summed histogram clearly shows features attributable to optical processes within the detector, for example reflections and scattering. These different processes can be separated by counting hits within certain time windows within the histogram. These same features are not evident in the single channel histogram, but they are still there, and the number of hits within the same time windows can be counted. In this way, we can decompose time histograms, even with low single channel statistics, such that the statistics will be significant when using all the PMTs. The time histograms will be analyzed on this tube-by-tube basis since when the laserball is at any other position the tube times are out of phase and the summed histogram is washed out. Note that due to computer time the histograms in figure 4 contain less than 1/5th of the statistics that will be collected during calibration.

To begin, considering equation (2) one wants to count the photons that arrived on time, i.e. no scattering or reflections. This is referred to as the prompt peak, which contains most of the light and is readily observed even in the single channel histogram. Now we need only to decide the integration limits around the peak. This is not a trivial issue since most scattering processes are strongly forward peaked and contribute to the prompt light. This is discussed in detail in sections 6 and 7. After extracting NHIT from all the histograms, and with a knowledge of the detector geometry we are now ready to find the optical parameters.

To get a feel for the scope of the problem one could consider moving the laserball to several locations and collecting NHIT for direct light incident on the PMTs. A linear minimum chi-squared fit might then proceed by considering the log of the above expression and inverting a $128 \times 128$ matrix (for 128 acrylic panels plus two extinction coefficients and a gain offset). Unfortunately, drifts in PMT efficiencies, uncertainty in the laserball pattern and covariances between the extinction coefficients make this method all but impossible. Strategies for fitting the optical parameters are discussed in section 6.
3 Hardware and Trigger

3.1 The Laserball

The main optical source is the nitrogen/dye laser with a diffuse ball and fibreoptic system (Figure 5); see [2] for a complete system description. The diffuse ball (called the laserball) can be deployed at various locations within the D2O volume by the rope manipulator system. The laserball can also be positioned on a single axis by lowering down through the acrylic vessel neck or through the calibration entry ports into the PSUP light water.

The primary laser source is a TEA thyratron triggered N2 laser producing a 900picosecond pulse at 337.1nm at a peak power of about 130kW. This primary beam can be delivered directly to the fibre-optic coupling optics, or it can be used to pump one of four available dye-laser units. The selection is made with a fully remote computer controlled movable mirror mechanism.

The dye laser units produce super-radiant incoherent radiation with approximately 300picosecond pulse width and about a 2nm bandwidth (FWHM). The present choices for the central wavelengths are 365nm, 386nm, 462nm and 420nm. Many other frequencies are possible; requiring the manual inter-change of a 5cc dye cuvette. Beam steering optics direct all the dye output to the same fibre-optic mixer.

The laser output stage consists of two remote independent neutral density filter wheels, fibre coupling stage, mode mixing yoke and PIN diode energy monitor.

The laser light is coupled into a bundle of 22 improved bandwidth, step index high OH silica fibres. This bundle is then cabled with a RG-174U transmission cable using loose irradiated PVC (heat shrink). This cable is then spliced with a soft silicone dielectric gel inside a 1/2" OD silicone tubing. This cable tubing (the source umbilical) is wound over an enclosed pulley array (like a block-and-tackle rig) which provides cable storage and tension control. The pulley array device is required due to the convoluted nature of this rotating coupler that would be required by any other cable take-up method.

The fibreoptic umbilical is terminated with a 10cm diameter diffuse ball which serves to distribute light isotropically through the detector. The diffuse ball consists of foamed glass bubbles mixed into a soft transparent silicone cured within a quartz glass boiling flask. The neck plug contains a fibreoptic position guide tube and also houses a miniature monitoring port (Ham-
Figure 9: Schematic of the N$_2$ laser calibration system.
3.2 Trigger and Histograms

The optical calibration requires post hit time histograms as described in section 2.3. The required time is the time difference between the laser light pulse and the arrival time of the proton pulse at the front-end card (FEC) after correcting for relative timing offsets. Remaining time offsets are constant and arbitrary, depending on cable lengths and speed of the trigger logic etc.

The laser system light pulse fires a fast node trigger that runs through a cable delay to a special FEC. The special FEC starts it’s own TAC and causes a global detector trigger to be set on the next 50MHz clock signal. The laser trigger delay will be set so that most of the approximately 250msec trigger window is utilized. Note that there should be no noticeable effect due to latching the trigger on to the 50MHz DFS clock since running up the laser trigger TAC will compensate for the extra time on the proton TACs.\(^3\) This can be verified by collecting data in event mode where all the TAC amplitudes are written to tape.

It is planned for the DAQ to be able to collect single channel histograms in one of two modes: event mode and histogram mode.

In event mode the DAQ event builder writes the laser events to tape exactly like a neutrino or other event. The event is identified as a laserball event as the global trigger was fired from the special FEC. Histograms are then filled in SNOMAN using a processor called the data filter. There are also several pusses from energy monitors in the laser system that go to other special FECs. Since these pusses are quite slow the DAQ will write them out some time later as a separate event record. Other computers, for example the calibration computer, can query the DAQ for the cumulative histogram of these monitor pulses.

In histogram mode the time differences are stored in histograms on the

\(^3\)This is called the synchronous trigger. Although an asynchronous trigger has been extensively discussed it is not known at this time if it will be implemented.
Figure 6: Single axis laserdial scans with two opposite facing pmts with pulse heights normalized to the laserdial internal pmt pulseheight. Angles for pmt 3 are displaced 180 degrees. The fibre-optic position in the laserdial was adjusted between the two scans to demonstrate how the isotropy can be adjusted. (a) Scan before fibre adjustment, (b) scan after adjustment.
front end embedded CPUs. The laser monitor pulses are histogrammed by
the DAQ. Events due to triggers from the laser FEC are not build and written
to tape. At this time it is not specified whether the histograms will be 1D
or 2D (time and charge), or whether the two TACs and the low and high
gain integrators will be separately histogrammed. Note that in this mode
the detector treats all other triggers in the usual way. One consequence of
this is that the detector will retrigger on late laser light. There may be
implemented a lockout on the NHT trigger following the laser pulse (eg.
400 nsec) to prevent these re-triggers. If we wanted to collect this late light
in a useful way the laser FEC would have to provide a retrigger after a set
time delay (eg. ~ 200 nsec).

4 Distances in Media

To fit an expression for NHT with the laserball (eg. equation 2) requires
calculation of the distances through each medium for every port and laserball
position. While this sounds straight forward there are subtleties involved.
One cannot simply take the straight line distances between the source and
the port because this would always reduce the distance in the acrylic (due to
refraction), resulting in a bad fit and a large error in the acrylic attenuation.

Even with simple geometry (an acrylic sphere) an analytic solution for
the path length is difficult. A reasonable approximation for small angles is
to assume a path in the D2O toward the port, refract through the acrylic
and then compensate the exit angle by simply joining to the port. A proper
solution can be found by iteration, where subsequent direction cosines are
estimated from the closest approach to a target port. The path distances in
each media are recorded after convergence to the target port.

The iteration method is able to cope with complex geometry but it is
slow and will fail to converge for (real) pathological circumstances. The
approximation is quicker and could include correction factors based on the
iterated calculation. For real time optical calibration it would be best to
have all the distances pre-calculated in a look-up table for 10 to 20 pre-
determined source locations. This would not be unreasonable as the optical
calibration would be executed by running the laserball at set locations within
the detector.

Another item to consider is what co-ordinate one takes for the location
of the pmt. What is required is the location that the photons entered the pmt concentrator bucket. Of course it is impossible to know this information and the hits in the cumulative histogram would have occurred all over the photo-cathode surface. We want a single target point that well represents the pmt in an average sense. The average must correctly weight the detection probabilities so that the resulting path distances are representative.

The problem can be significant for finding the correct path distance through the acrylic; it is much less important for the D_{2}O and H_{2}O distances. Consider figure (7), if we take the given pmt position (eg. from the LBL data) as the path target then there is an asymmetry in the detection probability around this point. This would cause a systematic under-estimate of the acrylic path length. A better estimate for the average target point is a point on the pmt axis about one centimeter outside the opening of the reflector bucket. The extra centimeter should be added back to calculate the solid angle correctly and an extra 10 centimeters should be added to estimate the H_{2}O path distance.

Really what we want is an average path through the acrylic weighted by the probability of a pmt hit, i.e.

$$F_{av} = \frac{1}{\Omega_{o}} \int x_{av}(\theta, \phi) P(\theta, \phi) \, d\Omega$$

(3)

where $P(\theta, \phi)$ is the probability of the pmt detecting a hit for a given path, $x_{av}(\theta, \phi)$ is the corresponding path length through the acrylic and $\Omega_{o}$ is the projected pmt solid angle. The SNOMAN Monte Carlo is actually well suited for evaluating such an integral. By running many events with an isotropic source we can weight $x_{av}$ with the number of hits. This calculation would have to be done off-line requiring the look-up table method for path distances in the optical parameter fitting.

A problem with optical calibration is that for positions in the D_{2}O the path lengths through the acrylic and H_{2}O are highly correlated, as seen in figure 8. Thus it takes high statistics and positions near the acrylic to effectively separate the H_{2}O and the acrylic attenuations. This degeneracy breaks only at the highest angles into the acrylic, precisely where the errors in the calculated distances will be greatest.
Figure 7: Defining the target distance to a pmt.
Figure 8: (a) Ratio of the distance in H₂O to the distance in acrylic as a function of path length in the D₂O for two laserball positions. This shows that the path lengths are highly correlated so that a determination of the H₂O attenuation is strongly coupled to the acrylic attenuation. (b) Ratio of the distance in D₂O to the distance in acrylic as a function of path length in the D₂O for two laserball positions. These attenuations are not strongly coupled.
5 The Time Histogram

In this section we examine closely the prompt time histograms and discuss how optical processes within the detector are manifest in the time distribution. We will then discuss how the various cuts are made on the time histograms to extract NHITs to fit for the optical parameters.

The time histogram (e.g., figure 4 and 10a) contains peaks due to the prompt laser pulse and any other focussed reflections. The remainder of the histogram is due to bulk scattering and unfocussed reflections. As discussed in section 2.3, the cumulative histogram would be smeared out for all other laserball positions. However, this histogram is still relevant as the similar features could be found in the individual pmt histograms with much higher statistics.

The time histograms are characterized by a large prompt peak with secondary peaks and scattering tails at later times. The largest secondary peak is found at 113.3 nsec which corresponds to the expected three radon transit time. The two earlier peaks are also due to pmt reflections, and are the result of the photocathode-reflector response which is discussed in section 8. SNO-MAN and the Queen’s Monte Carlo both include only Rayleigh scattering which produces the terraced features in the histogram. See figure (23) in section 7.1 where dipole scattering is discussed. Secondary scattering and reflection hits run down into the background noise hits before the TAC timeout.

The time histogram pattern for some different combinations of scattering and reflections can be studied from figure 9 where QMC data is presented with several conditional cuts.

Some individual pmt time histograms are shown in figure (10) for the laserball at several positions. Histogram (c) for PMT 9365 is an example of a lensing phenomena which is quite rare but it is expected from the detector geometry and must not be mistaken for prompt light. Histogram (d) shows a common problem with a light source in the H2O where broad peaks from reflections off the acrylic dominates the spectrum. Other effects due to pre- and after pulsing are not yet included in the Monte Carlo.

The optical analysis proceeds by locating the prompt peak with a sliding window algorithm. The algorithm finds the window location with the largest NHIT and checks if the peak is reasonably distinguished and close to the expected prompt time. Alternate peaks are inspected until the prompt light is identified. The peak is integrated to find NHIT and the centroid and
Figure 9: (a) All hits (b) No noise (c) No noise or scattering (d) No noise, scattering or AV reflections (e) No noise, scattering or omega reflections (f) Photocathode reflections only (g) Scattering only (h) Omega reflections only (i) AV reflections only (j) AV and omega reflections only
Figure 10: a) Summed time histograms for SNOMAN and QMC
b) PMT 1 for laserball at (0,0,0)
c) PMT 9365 for laserball at (0,0,-500)
d) PMT 50 for laserball in H₂O at (550,0,374)
widths are calculated from the distribution moments. The window width is set empirically at 7 nsec (see figure 15) due to the scattering tail on the late side of the prompt light. A possible refinement here is to double the integral of a half window on the early side of the peak centroid.

Other intervals relative to the prompt peak are then analyzed. A 30 nsec window following the prompt time is integrated to find the scattering contribution. The scattering component is characterized by the slope at the centre of the 30 nsec window; this is discussed further in section 7. Other time windows can also be integrated at this time, for example to examine the pmt reflectors using the 35° image ring (see section 8).

A remaining consideration is the adjustment of light source intensity to maintain single-photoelectron (spe) pmt response. At low intensities the photoelectron multiplicity is a Poisson distribution \( P(n, \mu) \) with a mean detected light intensity \( \mu \). If the laser is pulsing at rate \( R \) then in time \( t \) the number of pmt hits is

\[
\text{NHIT} = R t (1 - P(0, \mu)) = R t \left(1 - e^{-\mu} \right)
\]

(4)

To keep the probability of two or more photoelectrons so less than 1% would require that the mean event probability be less than 5%-3% everywhere. This would be impractical given the large variation in pmt-solid angles when moving the laserball, and would result in very low statistics. Instead we choose the intensity so that most of the pmts are at spe intensity and make the correction

\[
\text{Ef. Hits} = -R t \times \log \left(1 - \frac{\text{NHIT}}{R t} \right)
\]

(5)

where \( R t \) is the total number of laser flashes.

Higher photoelectron multiplicities will also distort the time spectrum and so it is important to understand this quantitatively. The time spectrum distortion is due to the discriminator firing on the leading edge of the pulse which results from the first photoelectron. The timing spectrum can be calculated by multiple convolution with a Gaussian for each photoelectron with a cut-off at the time for the first photoelectron. These spectra can then be summed with Poisson weightings. This calculation is easiest to perform as a Monte Carlo, since the first photoelectron time selection imposes awkward cut-offs on the integrals.

Figure 11 shows the results of such a calculation assuming a Gaussian shape with a sigma of 0.9 nsec for the laserball pulse and a pmt transit
time spread of 1.4 nsec. In the first plot the abscissa is the mean light intensity which is linearly related to the laserball output and equal to NHIT probability in the low intensity limit. In the second plot the abscissa is NHIT probability which would be just NHIT/Ft. The first line plotted is the ratio of the mean intensity to NHIT probability which shows how NHIT varies with multiple photoelectrons. The second line is the centroid value of the time peak (with an arbitrary 3 nsec offset) which is seen to shift to earlier times with increasing NHIT. From the third line we see that the sigma also decreases but only very slowly. From linear fits to the plots the following approximate adjustments can be used for NHIT probabilities up to 50% 

\[
t = t - 0.66 \times \frac{\text{NHIT}}{\text{Laser pulses}} \quad (6)
\]

\[
\sigma = \sigma - 0.042 \times \frac{\text{NHIT}}{\text{Laser pulses}} \quad (7)
\]

A spline to the data can be used for larger probabilities.

Note that the timing issues discussed here are in addition to the electronics Q versus T walk corrections. Since larger pulses will cross the discriminator threshold at earlier times there is a pulse-to-pulse systematic charge dependent shift. This correction is made by the crate level computers using look-up tables before the data is histogrammed or recorded to tape.

6 Fitting for Extinctions

6.1 D2O Attenuation

Although we expect to calculate posterior probabilities simultaneously from the global likelihood, it is a good idea to first start by just finding the attenuations separately. For the D2O a straight line fit for the attenuation ought to work well since large variations in the D2O path length can be achieved between source locations with small variation in other parameters (eg PMT angles). In particular one can think of the special case where the PMTs below the south pole view the laserball at several locations on axis.

The logarhythm of prompt NHIT, with solid angle and rate corrections is plotted in figure 12 for 105 PMTs that are viewed through the southpole panel at five east-axis laserball positions. A straight line fit yields the extinction coefficient (-0.000297 cm⁻¹ or 34m) and a gain offset which is quite
Figure 11: Plot of time centroid shift and sigma versus mean light and NHT Probability for a single pmt (see text).
arbitrary as the absolute QE is unknown (QMC Monte Carlo input value for the extinction coefficient is -0.00031 cm\(^{-1}\)).

Fitting with all the pmts offers much better statistics, but does introduce a dependence on other estimated parameters, ie. the acrylic attenuation and the angular pmt response. Figures 13 and 14 present results for QMC (-0.000256 cm\(^{-1}\)) and SNOMAN (-0.000404 cm\(^{-1}\)), SNOMAN Monte Carlo input value is -0.000497 cm\(^{-1}\)). The upper plots show a two dimensional chi-squared surface, with it's contour map, while the lower plot shows the best straight fit through the data.

Using this data I now return to the issue of choosing the width of the integration window for the time histograms. The above fit was repeated with NHT extracted using varying integration window widths. The result is shown in figure 15; empirically 7 usec is a reasonable choice for the integration width.

6.2 Acrylic Attenuation

Similar fits for the attenuation in the acrylic are shown in figures 16 and 17. Before fitting cuts are made on those pmts with large incidence angles ( > 40° ) since this light must have been scattered and because the angular response is less certain at larger angles. Cuts are made also for optical paths with a large distance in the acrylic since these path distances and directions are less certain. In future a statistical weighting scheme will be developed for these data since a large portion of the hits occur at large incidence angles. Fit results are 0.0201 cm\(^{-1}\) in QMC and 0.0182 cm\(^{-1}\) in SNOMAN for input value of -0.02 cm\(^{-1}\).

These fits to the acrylic attenuation draw attention to the problem of sensitivity to the pmt angular response. Figures 18, 19 and 20 show how most of the pmt hits always come from the pmts for which the incident angle for the direct line of sight is the largest. This means that most of the statistical weight is placed on high incidence angle hits, so that any uncertainty in the pmt angular response will cause errors in the fitted attenuations. The pmt angular response has been measured and is approximately flat out to 40° [9]. To investigate how the complicated optics of SNO may distort the large angle response I have reconstructed the effective pmt response from SNOMAN Monte Carlo data. I reconstruct the response by correcting NHT with the known attenuations and solid angles. The reconstructed response
Figure 12: Fit to $\log(NHT)$ and distance in the D$_2$O from QMC data with pms below the 111 panel (southpole) for the laser-ball at 5 positions on axis within the D$_2$O. Fit yields an extinction coefficient of -0.000297 (1/cm) or 34 m. The upper plot shows the Chi-squared surface.
Figure 13: Fit to log(NHIT) and distance in the D₂O from QMC data for the laserball at 0 positions on axis within the D₂O. Fit yields an extinction coefficient of $0.000286 \text{ (1/cm)}$ or 35m. The upper plot shows the Chi-squared surface.
Figure 14: Fit to log(NHIT) and distance in the D_2O from SNO-MAN data for the laserball at 5 positions on axis within the D_2O. Fit yields an extinction coefficient of -0.000464 (1/cm) or 20m. The upper plot shows the Chi-squared surface.
Figure 15: Fitted D\textsubscript{2}O attenuation and gain versus window width using SNOWAN data for southpole nmts. A window width of 7 ns appears to be reasonable choice.
Figure 16: Fit to log(NHIT) vs distance in the Acrylic from QMC data for the laserball at 5 positions on axis within the D_2O. Fit yields an extinction coefficient of 0.0091 (1/cm) or 50cm. The upper plot shows the Chi-squared surface.
Figure 17: Fit to log(NHIT) and distance in the Acrylic from SNO-MAN data for the laserball at 3 positions on axis within the D$_2$O. Fit yields an extinction coefficient of -0.0182 (1/cm) or 51cm. The upper plot shows the Chi-squared surface.
(figures 19 and 30) is nearly flat, with a slight downward slope, but also shows distortions, for example as 25°-30° with the laserball at (0,0,-500). Sensitivity to this distortions can be reduced by using a weighting inversely proportional to the pmt geometric distribution for each laserball position. An alternative is to model the pmt response as \( y = \alpha + \beta \theta \) where \( \alpha \) is absorbed in the overall gain and the slope parameter \( \beta \) is a free parameter fitted with the data (the same for all pmts).

6.3 Simultaneous Fit

A multi-dimensional linear function (equation 2) is minimized for three parameters, the \( D_2O \) and acrylic attenuations and a gain figure. The chi-squared surfaces are shown in figures 21 and 22 for QMC and SNOMAN data. The best value for QMC are \(-0.000284 \text{ cm}^{-1}\) for \( D_2O \) and \(-0.0172 \text{ cm}^{-1}\) for acrylic. The input values are \(-0.00031 \text{ cm}^{-1}\) and \(-0.02 \text{ cm}^{-1}\). For SNOMAN the fitted values are \(-0.00406 \text{ cm}^{-1}\) for \( D_2O \) and \(-0.0200 \text{ cm}^{-1}\) for acrylic, while the input values are \(-0.00407 \text{ cm}^{-1}\) and \(-0.02 \text{ cm}^{-1}\).

7 Fitting for Scattering

Light scattering in the SNO detector is a difficult phenomenon to measure as it can have many causes and effects. Scattering can be intrinsic to the material, like Raleigh scattering, or can be due to coherent or incoherent scattering and reflections off particulate matter ranging in size from nanometers to tens of microns. Due to the large size of the SNO detector, even in infinitely clean water, scattering is a common process with over 10% of detected light having been scattered. Scattering can change the resolution and gain of the detector. Fitters that attempt to identify events by hit pattern recognition might be particularly sensitive to scattering distributions that are different from any training sets used.

Instead of attempting to identify and measure all the scattering processes we intend to quantify the effects of scattering guided by how it impacts the interpretation of the SNO data. Any light scattering will cause pmt hits to be out-of-time and out-of-position in a non-random way (unlike noise hits). A pattern recognition fitter wants to know how out-of-time and how out-of-position the hits are; knowing a total scattering cross-section is much less
Figure 12: The top plot is the angular distribution of incident photons at each pmt in QMC for the laserball at (0,3,-500). The second plot is the angular distribution of detected photons. The third plot is proportional to the ratio which is the pmt response.
Figure 19: (a) Plot of the angular distribution for the line of sight to each pmt in SNOMAN from a point at (0,0,500). (b) The angular distribution weight by NHT. (c) Calculated pmt response by dividing out the attenuations and solid angle. (d) Effective pmt response by making a weighted histogram from plot (c) and dividing out the geometric distribution (plot a).
Figure 2b: (a) Plot of the angular distribution for the line of sight to each pmt in SNOMAN from a point at (0,0,250). (b) The angular distribution weighted by NHIT. (c) Calculated pmt response by dividing out the attenuations and solid angle. (d) Effective pmt response by making a weighted histogram from plot (c) and dividing out the geometric distribution (plot a).
Figure 21: Chi-squared over variations in D$_2$O attenuation and acrylic attenuation from QMC data for five laserball positions in the D$_2$O. Best fit yields -0.000284 for D$_2$O (35m) and -0.0172 for acrylic (58cm).
Figure 22: Chi-squared over variations in D_2O attenuation and acrylic attenuation from SNOMAN data for five laserball positions in the D_2O. Best fit yields -0.306406 for D_2O (25m) and -0.0286 for acrylic (36cm).
useful.

We intend to quantify bulk scattering in SNO with several parameterizations. Working from the time histograms we concentrate on the height and shape of the hits in set time windows following the prompt light. From this we can define an effective scattering cross-section. This cross-section will be smaller than the real cross-section as it will ignore forward scattered light. This is sensible as the SNO data cannot distinguish forward scattered or phase shifted light. The shape of the time distribution, parameterized by the slope at the window midpoint can indicate the broad nature of the angular distributions. Again this is reasonable as there will never be enough detector event data to be sensitive to any fine structure of coherent scattering angular distributions.

It is possible, and it is planned, in SNO to study scattering distributions using a directed light source. In this report I discuss only scattering with an isotropic source, as this data will be embedded within the calibration histograms.

There are three modes of scattering to be considered: dipole (Rayleigh) scattering, Mie scattering and scattering from larger particulate matter. Rayleigh scattering is due to thermally fluctuating electron densities; it is expected to dominate due to the wide $(1 + \cos^2 \theta)$ angular distribution.

Mie scattering is the intermediate case where the size of the scattering centre is similar to the photon wavelength. The angular distributions must be worked out more rigorously due to phase coherences. The effect is to produce angular distributions that are very forward peaked, so that in the SNO detector the result is better characterized as an absorption.

Any larger particulate matter suspended in the water will absorb and reflect light. Assuming a spherical shape of radius $a$, the reflected light is uniform

$$\frac{dN}{d\Omega} = \frac{a^2}{4} \tag{8}$$

Since the distributions will be indistinguishable, I combine specular reflection with Rayleigh scattering by considering the distribution

$$\frac{dN}{d\Omega} = 1 + k \cos^2(\theta) \tag{9}$$

where $0 \leq k \leq 1$. 38
I study each of these scattering processes in the following sections. The conclusion is that for the purposes of energy calibration and fitting all bulk scattering can be characterized by a single effective scattering length and absorption length.

7.1 Dipole and Specular Scattering

To study the effects of scattering I start with dipole (Rayleigh) scattering, then try varying distributions keeping with the same general form. For the predicted water purity level then Rayleigh scattering will be the dominant process. For an optical source at the center of the detector there is a time period of about 40 ns. In which there can be no real track hits unless the photon has made a large angle scatter from within the bulk medium. This suggests an opportunity for determining a characteristic scattering length for dipole scattering in water. The experimental data is unreliable in this area, but since dipole scattering is due to variations in electron density then the scattering should be the same in D₂O and H₂O.

Averaged over polarisations the dipole angular distribution has a \((1 + \cos^2 \theta)\) shape. To permit an admixture of bulk specular scattering I take the distribution \((1 + k \cos^2 \theta)\). Including absorption and transforming the angles into time one finds, for the laser at the centre of a homogeneous detector, the time spectrum for photons undergoing a single Rayleigh scattering event is

\[
\frac{dN}{dt} = \frac{3N_0}{2k + 3} \exp \left( -\frac{vt}{x_a} \right) \int_0^{\infty} \left( 1 + k \left( \frac{R^2 + x^2 - R^2}{2vt^2} \right) \right)
\times \left( \frac{v^2 t^2 + R^2 - x^2}{2vt^2} \right) \exp \left( -\frac{x}{x_a} \right) \times \alpha(\cos \alpha) \, dt
\]

where \(\alpha(\cos \alpha)\) is the pmt efficiency and angular response with

\[
\cos \alpha = \frac{R^2 + x^2 - y^2}{2vtR}.
\]

The absorption and scattering lengths are given by \(x_a\) and \(x_s\). Figure (23) shows the results of a computation, where the integral was split into the D₂O and the H₂O regions and integrated across the detector several times.
Terms were included for specular reflection off the pints, re-transit across the detector and multiple scatters. The spectrum has unphysical sharp secondary peaks since the calculation assumes that all reflected light bounces back from the pints at 180°. I use this time distribution in all subsequent analyses since generating these data in the full Monte Carlo (SNOMAN) would take several CPU-weeks.

I now chose a window from 45 to 65 nsec and integrate the number of counts for calculated time spectra using a range of D$_2$O absorption lengths and scattering lengths. A 3D map of this integral versus the absorption length and scattering length is shown in figure 24 for $k = 0$ and $k = 1$. The calculation was done also for other k values but are not shown as these are consistent with figure 25. We see that for reasonable values of the absorption length (about 60m) the 45-65 nsec integral is fairly independent of the absorption and k. Thus for all manner of dipole and specular scattering the effective scattering length is easily parameterized by the 45-65 nsec integral.

7.2 Non-dipole Scattering

I now turn to the intermediate case where scattering is from foreign impurities in the water of a size comparable to the wavelength of light. This is known as Mie scattering. The problem rapidly becomes intractable unless some simplifying assumptions are made about the scattering object shape and composition. Waltham [10] considered scattering from the bacteria E.coli whose refractive index was measured to be $n_s = 1.415$. He found that the total scattering cross-sections calculated for an ellipsoidal shape agreed with measurements.

I now calculate the angular distributions for E.coli and then determine the hit time distribution in SNO for an isotropic source at the centre of the detector. The light intensity a distance r away from a scattering centre is

$$I = I_0 \frac{|S_1(\theta)|^2}{2k^2 r^2}$$  \hspace{1cm} (12)

where $k = 2\pi n_s / \lambda$ and

$$S_1(\theta) = \sum_{n=0}^{\infty} \frac{2n+1}{n(n+1)} \left\{ a_n r_n(\cos \theta) + b_n r_n(\cos \theta) \right\}$$
Figure 23. Comparison of simple calculated time spectrum to Monte Carlo data (QMC).
Figure 24: Integral of the 45-65 nsec window as a function of scattering length and absorption length for the distribution $1 + k \cos^2(\theta)$. For the upper plot $k = 0$ and for the lower plot $k = 1$. We see that the integral can be specified by an effective scattering length and is fairly independent of $k$ and the absorption.
Figure 25. The top plots show the polar and angular distribution for dipole-like scattering. The middle plots show the distribution calculated from Mie theory for several aspect ratios. The lower plots show a simple model for Mie type scattering distributions.
\[ S_{2}(\theta) = \sum_{n=0}^{\infty} \frac{2n+1}{n(n+1)} \left( b_{n} \tau_{n}(\cos \theta) + a_{n} \tau_{n}(\cos \theta) \right) \]  

(13)

The Bie functions \((\tau_{n})\) and coefficients \((a_{n}(z), b_{n}(z))\) can be found in [11] and are functions of the aspect ratio \(z = \kappa / a\) with \(a\) the particle radius. Some angular distributions are plotted in figure 25 for the range of aspect ratios of interest \((z = 4 \text{ to } z = 16)\).

Similar to equation 10 the time distribution is

\[
\frac{dN}{dt} = \frac{2\pi N_{0} R t c}{(f(\theta) z)} \exp \left( -\frac{\nu t}{x_{a}} - \frac{\nu t}{x_{c}} \right) \int_{0}^{\theta} \left( f \left( \frac{R_{0} - y^{2} - z^{2}}{2\nu t} \right) \right) \exp \left( -\frac{z}{x_{a}} - \frac{x}{x_{c}} \right) dx \, d\theta
\]

(14)

where the pmt angular response has been approximated by \(\cos \alpha\). The angular function \(f\) is given by

\[
f(\theta) = |S_{1}(\theta)|^{2} + |S_{2}(\theta)|^{2}
\]

(15)

but was actually replaced by a Chebyshev polynomial fit to the distributions. As before, equation 14 was integrated to produce a time spectrum, including terms for reflection off the pmts. Again the counts in the 45 to 65 msec window are summed. These results are shown in figure 26, where the integral counts in the 45-65 msec window is plotted as a function of the scattering particle aspect ratio. In the first case the scattering length is held fixed at \(x_{a} = 6666\)cm; we see clearly that the time distribution is insensitive to the aspect ratio (and hence insensitive to the details of the angular distribution). In the second case the bacterial density \(c^{3}\) is held fixed at 8524 cm\(^{-3}\) and the scattering cross-section is calculated from the Mie formula. The integral is linearly dependent on the aspect ratio, hence the time distribution is well characterized by a single parameter: the scattering length. That is, whether any change in scattering length is due to a change in particle (bacteria) size or a change in the bacterial density does not matter. It can noted that the bacterial density used here 8524 cm\(^{-3}\) is very high, yet still the resulting time

\[4\]
distribution integral is down by about an order of magnitude from Rayleigh scattering.

To investigate the sensitivity to the forward peaks in the angular distributions I replaced $f$ with a simple form with a forward peak parameterized by $\beta$ (see figure 25)

$$ f(\theta) = \frac{1}{1 + 10^{\beta \theta^2}} $$

Again calculating time distributions and integrating the 45-65 usec window the results are seen in figure 21. I plot the 45-65 usec window as a function of the parameter $\beta$. We see that when $\beta$ is negative we essentially have Rayleigh scattering. As $\beta$ gets to a value of 2 the distribution is becoming forward peaked as in Mie scattering and the integral drops rapidly. For $\beta$ larger than 2 the integral is constant with a value just the same as for Mie scattering with the same scattering length.

I conclude here that the SNO detector is sensitive only to wide angle scattering like Rayleigh or specular scattering and that this scattering can be parameterized by a single effective scattering length. Scattering then can always be assumed to be uniform with an effective scattering length given by the integral of the time histogram between 45 and 65 usec. Mie scattering has a very minor effect due to the strong forward peak of the angular distribution. In fact SNO can tolerate bacterial counts an order of magnitude higher than Waldman's initial estimates [$10^5$ L$^{-1}$] while still being below Rayleigh scattering. High bacterial counts will just add to the effective scattering length while the light distribution within the detector will still be well modeled by a uniform distribution with this effective scattering length.

8 PMT reflectors

In figure (10A) for a laserball at the centre we see there are several peaks in the spectrum associated with reflections off the pmts; in SNO-MAN there are peaks at 96.8 and 133.3 usec and in QMC at 91.5, 99.5 and 133.3 usec.

To explain these we consider the distances and reflection angles that would correspond to these peaks as a transit time offset. The peaks at 133.3 usec are clearly direct reflections equivalent to a triple radius transit. Taking the photon path to be a detector radius plus a PSUP cord, one finds the pmt
Figure 26: The upper plot is the time distribution for Mie scattering with various aspect ratios. The lower plot shows the counts in the 45-65 ns window versus the aspect ratio in Mie scattering for (a) constant density with the scattering length worked out from the aspect ratio, and (b) constant scattering length.
Figure 27: The upper plot is the time distribution for the scattering model $1/(1 + 10^6 \theta^2)$ with several values of $\beta$. The lower plot shows the counts in the 45-65 nsec window versus $\beta$. 
reflection angle to be given by
\[
\cos(\theta) = 0.01336 t - 0.5
\] (17)

Thus for SNOMAN there is a prominent reflection at about 37° while QMC has pmt reflections at 44° and 34°. A strong shoulder on the 47° peak in SNOMAN suggests it may have an enhanced reflection at about 47°. These peaks at roughly 35° and 45° must be due to light photocathode-reflector interactions that happen to be enhanced at these angles.

Tracing possible ray paths for the pmt-reflector combination I find that the wide 45° reflection requires a large angle (approx 50°) reflection from the photocathode. This is perhaps not so surprising since it is well known [8] that the photocathode reflection coefficient rises dramatically at 50° for the parallel polarization. Since this reflection is near the bucket cutoff then the slightly different reflector shapes for SNOMAN and QMC might explain why the peak is only prominent in QMC.

The peak at 35° has no similar explanation and must be due to a more subtle geometry arrangement. As an experimental curiosity I positioned a SNO pmt/reflector unit on a bench pointed toward a wall at a distance of 230cm. Slicing a beam flash light toward the pmt and along the axis produced reflections projected on to the wall. A sharp image of the reflector petals was observed in a ring with 160cm radius on the wall, corresponding to a reflection angle of 35.5°. It was quite surprising that the non-imaging reflectors when combined with the photocathode do produce an image at a certain angle and that this image was predicted by the Monte Carlo. This is a good boost of confidence for the pmt model since this reflection had not been noticed before it was uncovered by Monte Carlo data.

To reflect off at 35°, the photons must bounce only once off both the photocathode and the omega reflector. The number of hits under the 35° peak relative to the prompt peak provide a good way of making a global evaluation of the omega reflectivity. As a quick evaluation of this proposal I ran SNOMAN several times with a reduced value for the omega reflectivity coefficient. Figure 28 is a plot of the ratio of the 35° peak to prompt peak. This ratio is actually a bit complicated since both the reflecting and the hit pmt has a dependence on the reflector. The approximation here is that the 35° peak varies with the coefficient squared, while the prompt peak is partially linearly dependent on the reflection coefficient. It seems like it

48
would be straightforward to track the integrated reflector condition from this ratio. Note that this ratio would also have a dependence on the D$_2$O attenuation length that should be accounted for.

9 On-going and Future Work

In this report I have attempted to present the methodology that is being developed for the optical calibration. This report is meant to provide the background for future reports on specific analysis. Specifically I have not included in this report the fitting for the acrylic panel-by-panel or an uncertainty analysis.

The software routines are now under construction to perform a complete optical fit with uncertainty analysis. The final stage of this analysis is then to construct and test a detector response function $E = E_{\text{hit}}(n, R, \theta)$ based on the fitted optical constants.

SNOMAN requires changes to permit variations in transmission in acrylic from panel to panel and variations in individual points (e.g., the QE's distributed according to [12]).

Further changes are also being made to the optical hardware, including development of a shortwave length source and directional/multi-directional fittings for the laserball. I am also developing a driver circuit for a double heterostructure UV diode. These will be described in [2].

References


Figure 28: Corrected ratio of hits in the 35° peak to the prompt peak plotted versus the prompt omega reflectivity.


