

The Impedance for Water Flow through the Photomultiplier Sphere

by N.W. Tanner

SNO-STR-92-024

1. Introduction

It is extremely desirable to achieve a light water flow pattern through the photomultiplier sphere which is everywhere radially outward to avoid the risk of contamination, radio-active and optical, from the relatively dirty components outside the sphere. To do so it is necessary to obtain an over-pressure within the sphere which is large enough to prevent the turbulent convection current outside the sphere from passing through the sphere.

In the ASC Report no.90-1022-1 it was estimated (with some uncertainty) from the preliminary turbulent convection calculations that a sufficient overpressure would be obtained with the recirculation flow of 135ℓ/min if the photomultiplier sphere blocked the flow to ~99.9%, - equivalent to an orifice of ~1cm² per photomultiplier (see Section 4.5 and Appendix B of the ASC Report). Subsequently the target for the flow impedance of the sphere was set at an orifice of 0.4cm² per photomultiplier.

The pressure drop Δp through an orifice, of diameter \gg thickness of material, is related to the velocity of flow V in the orifice by the Bernoulli equation $\Delta p = \frac{1}{2}\rho V^2$, and is approximately independent of the viscosity. For the recirculation flow of $Q_0 = 2.25 \times 10^{-7} \text{ m}^3/\text{s}$ and an orifice of 0.4cm², both per photomultiplier, $V = 5.6 \times 10^{-3} \text{ ms}^{-1}$ and $\Delta p = 0.0157 \text{ Pa}$. This pressure head Δp and the recirculation flow Q_0 are taken as the design parameters for the water tightness of the photomultiplier sphere. For impedance it is convenient to work in units of $Z_0 = \Delta p/Q_0$.

The flow apertures which occur between the photomultipliers, concentrators, hexagons and other components of the photomultiplier sphere are generally such that the better approximation is an aperture of linear dimension small c.f. the thickness of material. In this case the flow is controlled by the viscosity η and Q is proportional to Δp . For the purpose of making estimates of the flow Q on impedance $Z = \Delta p/Q$ it suffices to consider a few cases of simple geometry:

- (i) Circular pipe of length L and radius R ,

$$\begin{aligned} Q &= \pi R^4 \Delta p / 8 \eta L \\ &= 4.72 R^4 / L \quad \text{m}^3 \text{ s}^{-1} \end{aligned} \quad (I)$$

for $\Delta p = 0.0157 \text{ Pa}$ and $\eta = 1.307 \times 10^{-3} \text{ Pa.s}$ for water at 10°C.

- (ii) Long slit of gap x , length L in the flow direction, and width w ,

$$\begin{aligned} Q &= \Delta p w x^3 / 12 \eta L \\ &= 1.001 w x^3 / L \quad \text{m}^3 \text{ s}^{-1} \end{aligned} \quad (II)$$

(iii) Annular pipe of radii R_1 and $R_2 = R_1 + \Delta R$, and length L ,

$$\begin{aligned} Q &= \pi \Delta p \{ R_2^4 - R_1^4 - (R_2^2 - R_1^2)^2 / \ln(R_2/R_1) \} / 8\eta L \\ &\approx \pi \Delta p R_1 \Delta R^3 / 6\eta L \quad \text{for } \Delta R \ll R_1 \\ &= 6.29 R_1 \Delta R^3 / L \quad \text{m}^3 \text{ s}^{-1} \end{aligned} \quad (III)$$

2. Calculation of Flow Rates and Impedances

Fig.1 shows the general assembly of the photomultiplier, concentrator, and hexagon. It is of interest to consider the water flows and impedances **without** the use of the silicone rubber indicated in the figure, or elastomer gaskets.

(a) Air bleed holes

These are sets of 6 holes of 2 or 3mm diameter at the vertices of the hexagon and also in the concentrator disk (not shown). The thickness of the material is about 5mm. Using equation (I) gives:

$$\begin{aligned} R &= 1.0 \times 10^{-3} \text{ m} \quad L = 5 \times 10^{-3} \text{ m} \\ Q_2 &= 2.52 \times 10^{-2} Q_0 \text{ per 6 holes} \\ \text{and } Z_2 &= 39.7 Z_0 \end{aligned}$$

$$\begin{aligned} R &= 1.5 \times 10^{-3} \text{ m} \quad L = 5 \times 10^{-3} \text{ m} \\ Q_3 &= 0.128 Q_0 \text{ per 6 holes} \\ Z_3 &= 7.84 Z_0 \text{ per 6 holes} \end{aligned}$$

(b) Front rim of concentrator/hexagon

The concentrator is free to move in the axial direction relative to the PMT-baffle and is held in place by the spring loading on the photomultiplier which is indicated schematically in Fig.1. The net load on the concentrator, allowing for buoyancy, might be chosen to fall in the range 2 to 10 Kg, depending on the orientation of the hexagon. This is a modest load and the magnitude of the effective gap, x in equation (II), between the rim of the concentrator and the front of the hexagon will depend on the flatness of the injection moulded plastic components and the possible use of webs at the vertices of the hexagon to stiffen the front. For the purpose of making an estimate it will be assumed $\sqrt[3]{\langle x^3 \rangle} = 0.5 \text{ mm}$, $L = 3 \text{ mm}$, and $w = 860 \text{ mm}$ in equation (II) giving

$$\begin{aligned} Q_F &= 0.159 Q_0 \\ \text{and } Z_F &= 0.28 Z_0 \end{aligned}$$

with an uncertainty of something like a factor of 10.

(c) Concentrator/PMT Baffle

This is a cylindrical sliding joint for which the flow is given by equation (III) with $\Delta R = 0.25\text{mm}$ (maximum), $R_1 = 111\text{mm}$, and $L = 15\text{mm}$ (nominal). This gives

$$Q_{CB} = 3.2 \times 10^{-3} Q_0$$

$$\text{and } Z_{CB} = 309 Z_0$$

(d) PMT Baffle/Hexagon

The periphery of the hexagonal flange of the PMT Baffle has a length $w = 990\text{mm}$ and a thickness $L = 5\text{mm}$. The effective gap will probably be determined by the distortion of the hexagons under load but $x = 0.5\text{mm}$ might be a plausible guess. Then from equation (II)

$$Q_{BH} = 0.110 Q_0$$

$$Z_{BH} = 9.09 Z_0$$

(e) Hexagon/Hexagon

The periphery of the hexagon is $w = 1024\text{mm}$ and the length $L = 150\text{mm}$. For an effective gap $x = 1\text{mm}$ between adjacent hexagons, the half flow which can be assigned to one photomultiplier unit is given by equation (II) as

$$Q_{HH} = 3.04 \times 10^{-2} Q_0$$

$$Z_{HH} = 32.9 Z_0$$

(f) Concentrator/Photomultiplier

Where the photomultiplier is in contact with the concentrator the photomultiplier surface has a hoop radius of 98mm and a meridional radius of 59.4mm . The difference of maximum and minimum diameter of the photomultiplier at the concentrator location has been measured at Queen's U. and LBL for 60 photomultiplier units with similar results: Queen's $0.064 \pm 0.030\text{mm}$; LBL $0.069 \pm 0.052\text{mm}$; (mean and standard deviation). The mating surface of the concentrator is a machined cone of half angle 20° the surface of which is tangential to the photomultiplier and has a length of 4mm . Treating the concentrator/photomultiplier as a circle circumscribing an ellipse and averaging over the geometry and the statistical distribution gives an effective gap perpendicular to the photomultiplier surface of $\sqrt{\langle r^2 \rangle} = 0.0275\text{mm}$ in equation (II), with $L = 4\text{mm}$ and $W = 616\text{mm}$.

$$Q_{CP} = 1.42 \times 10^{-5} Q_0$$

$$Z_{CP} = 7.04 \times 10^4 Z_0$$

It is clear that the eccentricity of the measured photomultiplier is a negligible consideration. The actual gap could become larger because of distortion of the plastic consequent on the relaxation of the moulding stresses after machining, but the geometry and loading are very favourable and it seems unlikely that

Q would exceed one per cent of Q_0 , and we shall adopt $Q_{CP} \leq 10^{-2} Q_0$, $Z_{CP} \geq 100 Z_0$.

3. The Net Impedance to Flow

From Fig.1 total flow impedance per photomultiplier amounts to

$$Z_T = \left\{ \left[(Z_3^{-1} + Z_F^{-1} \parallel Z_2^{-1})^{-1} + (Z_{BH}^{-1} + Z_{CB}^{-1} + Z_2^{-1})^{-1} \right]^{-1} \right. \\ \left. Z_{HH}^{-1} \parallel Z_{CP}^{-1} \right\}^{-1}$$

with $Z_2 = 39.7 Z_0$

$Z_3 = 7.84 Z_0$

$Z_F = 6.28 Z_0$ with an uncertainty of a factor of ~ 10 .

$Z_{CB} = 309 Z_0$

$Z_{BH} = 9.09 Z_0$

$Z_{HH} = 32.9 Z_0$

$Z_{CP} > 100 Z_0$

With the estimates as given $Z_T = 7.34 Z_0$. If Z_F is small then $Z_T = 5.59 Z_0$. In both cases Z_{BH} is the controlling impedance and if this were made large, by reducing π or increasing L or sealing with silicone rubber, Z_T could be made as large as $14.5 Z_0$ independent of Z_F .

4. Conclusions

- (i) The impedance for water flow through and between the hexagons could reasonably be made an order of magnitude higher than the effective impedance of a 40mm^2 orifice per photomultiplier.
- (ii) Whether a correspondingly high impedance can be achieved with the seals between the hexagon panels is something which LBL will want to consider.
- (iii) If an overall net impedance of $10 Z_0$ could be obtained this would raise the pressure head to 0.157 Pa and give a much more comfortable safety factor with respect to convection currents flowing inwards through the photomultiplier sphere.
- (iv) There is no case at all for sealing, with silicone rubber or elastomer, the concentrator to the photomultiplier or the PMT Baffle to the concentrator.
- (v) There is a case for improving the seal between the PMT Baffle and the hexagon, which is a matter which rests with LBL.
- (vi) It will not be possible to predict the impedance Z_F with any confidence until injection moulded prototypes are available.
- (vii) If Z_F is small c.f. Z_2 there will be very little flow through the concentrator dish. However even if Z_F is large it is still likely that the turbulent convection currents within the photomultiplier sphere will mix any leaching products from the concentrator and photomultiplier into the main body of water.

- (viii) It is quite impractical to simulate in the laboratory the flow conditions under the minute pressure head Δp . On the other hand it is easy to measure gaps of significant size, typically several tenths of a mm.
- (ix) There is a powerful case for mounting directional flow sensors on the PMT baffles inside the hexagons. How else will we know whether or not there is radially inwards flow?
- (x) The impedance of the bleed holes for air is a factor of 1.4×10^{-2} less than for water.

Fig. 1

